1 Omitted Calculation for Parametric Example

In this part of the appendix, we provide the detailed calculation behind how our reporting rule helps sustain efficient relational contract in the numerical example in Section 2. Recall that in the example, we set \( c = 1 \), \( y = 26 \), and \( p = 1/13 \). In addition, the minimum bonus to motivate the agent under truthful reporting is \( 13 \), and the cutoff discount factor \( \delta^* \) is \( 13/14 \).

Under the relational contract considered in the example, the principal pays the agent a fixed wage \( w^* \) and an additional \( b^* \) if the supervisor reports \( G \) in each period. Specifically, we set \( b^* = 13 \) and \( w^* = c - (4/13) b^* \). This implies that at \( \delta^* = 13/14 \), the principal’s discounted future surplus is equal to 13, so she is willing to pay the bonus. To show that our reporting rule lowers the cutoff discount factor, it therefore suffices to show that the agent has a strict incentive to exert effort, or equivalently, the agent’s payoff if he shirks is strictly lower than his equilibrium payoff.

As mentioned in the example, calculating the agent’s payoff once he shirks requires us to take into account the possibility of multistage deviations by the agent. Specifically, if the agent shirks and a good report is sent out, he knows that output is
0. This information may induce him to shirk again or even exit the relationship. To tackle the possibility of multi-stage deviation, notice that under our reporting rule, the agent’s expected discounted payoff is determined by the probability that output was \( y \) in the previous period—and if this is the first period of the game or if the report in the previous report was \( L \), we can set this probability to be \( 1/4 \). Now denote \( V(\rho) \) as the agent’s expected discounted payoff if output in the previous period was \( y \) with probability \( \rho \).

Along the equilibrium path, this probability is always \( 1/4 \), so the agent’s equilibrium payoff is given by

\[
V\left(\frac{1}{4}\right) = w^* - c + \frac{4}{13}b^* + \delta^* V\left(\frac{1}{4}\right),
\]

(V-A)

Now if the agent shirks, his payoff is given by

\[
w^* + \frac{1}{4} \left( b^* + \delta^* V(0) \right) + \frac{3}{4} \delta^* V\left(\frac{1}{4}\right),
\]

where notice if the agent shirks, the supervisor still sends a good report with probability \( 1/4 \), but when this occurs, the agent knows that output is \( y \) with probability 0, so his continuation payoff is \( V(0) \). Comparing these two expressions, we then obtain that the agent will not shirk if

\[
\frac{3}{52} b^* + \frac{1}{4} \delta^* \left( V\left(\frac{1}{4}\right) - V(0) \right) \geq c.
\]

Using \( b^* = 13 \), \( \delta^* = 13/14 \), \( c = 1 \), we can write the agent’s incentive compatibility condition as

\[
V\left(\frac{1}{4}\right) - V(0) \geq \frac{14}{13}.
\]

(IC-A)

We now show that this inequality is strict. For now, suppose that the agent will not exit the relationship following a deviation, and we will return to this issue later. Given that the agent will not exit, we only need to consider the agent’s subsequent effort choices. In this case, we have

\[
V(0) = \max\{V_e(0), V_s(0)\},
\]

where \( V_e(0) \) is the agent’s payoff if he works and \( V_s(0) \) is his payoff if he shirks.
Now
\[ V_e(0) = w^* - c + \frac{1}{13} (b^* + \delta^* V(1)) + \frac{12}{13} \delta^* V \left( \frac{1}{4} \right), \]
where notice that if the supervisor sends \( G \), the agent knows that output is \( y \) for sure, and therefore, his continuation payoff is \( V(1) \).

Similarly,
\[ V_s(0) = w^* + \delta^* V \left( \frac{1}{4} \right) \]
since the supervisor sends \( B \) for sure in this case.

Notice that if \( V(0) = V_s(0) \), we can then use (V-A) to show that
\[ V \left( \frac{1}{4} \right) - V(0) = \frac{4}{13} b^* - c = 3. \]
The inequality in (IC-A) is therefore strict, and we are done.

Now suppose \( V(0) = V_e(0) \) so that
\[ V(0) = w^* - c + \frac{1}{13} (b^* + \delta^* V(1)) + \frac{12}{13} \delta^* V \left( \frac{1}{4} \right). \]
Using (V-A), we then have
\[ V \left( \frac{1}{4} \right) - V(0) = \frac{3}{13} b^* - \frac{1}{13} \delta^* \left( V(1) - V \left( \frac{1}{4} \right) \right). \quad \text{(DIFF)} \]
This equality implies that to show that \( V \left( \frac{1}{4} \right) - V(0) > 14/13 \), we need to evaluate \( V(1) - V \left( \frac{1}{4} \right) \). Now \( V(1) = \max \{ V_e(1), V_s(1) \} \), so there are again two cases to consider.

First, suppose \( V(1) = V_e(1) \). In this case,
\[ V(1) = w^* - c + b^* + \delta^* V \left( \frac{1}{13} \right) \leq w^* - c + b^* + \delta^* V \left( \frac{1}{4} \right), \]
where the equality follows from the fact that the supervisor always reports \( G \) in this case so output is \( y \) with probability \( 1/13 \), and the inequality follows from the fact that the agent’s value is increasing in the probability that output is \( y \). This inequality, together with (V-A), then implies that
\[ V(1) - V \left( \frac{1}{4} \right) \leq \frac{9}{13} b^*. \]
Using (DIFF), we obtain that
\[ V\left(\frac{1}{4}\right) - V(0) \geq \frac{3}{13} b^* - \frac{1}{13} \delta^* \cdot \frac{9}{13} b^* = \frac{33}{14}. \]
It then follows that the inequality in (IC-A) is again strict in this case.

Second, suppose that \( V(1) = V_s(1) \), so that
\[ V(1) = w^* + (b^* + \delta^* V(0)). \]
This expression, together with (V-A), then implies that
\[ V(1) - V\left(\frac{1}{4}\right) = \frac{9}{13} b^* + c - \delta^* \left( V\left(\frac{1}{4}\right) - V(0) \right) \geq \frac{9}{13} b^* + c = 10. \]
Now using (DIFF) we have
\[ V\left(\frac{1}{4}\right) - V(0) > \frac{3}{13} b^* - \frac{1}{13} \delta^* 10 = \frac{16}{7}, \]
so that the inequality in (IC-A) is also strict in this case. Combining all cases, we have shown that the agent’s IC constraint is slack as long as he does not exit.

Finally, we need to show the agent will not exit the relationship. To ensure this, the timing of bonus payment is important. As in our general analysis, we assume that the principal pays the bonus at the beginning of the next period as part of the fixed wage rather than at the end of the current period. Notice that the principal is indifferent between paying \( b^* = 13 \) at the end of current period and paying \( \frac{b^*}{\delta^*} = 14 \) at the beginning of next period. But since the bonus is postponed to after the agent accepts next period’s contract, the agent is less likely to exit. To see that the agent will not exit with this timing of bonus payment, notice that if the supervisor sends \( B \), the agent’s continuation payoff is \( V(1/4) = 0 \), so he will not exit. And if the supervisor sends \( G \), the agent will not exit as long as \( \frac{b^*}{\delta^*} + V(0) \geq 0 \). Our analysis above implies that
\[ b^* + \delta^* V(0) \geq b^* + \delta^* V_s(0) = b^* + \delta^* \left( c - \frac{4}{13} b^* \right) = \frac{143}{14} > 0, \]
so that the agent again will not exit. This finishes the proof that \( V(1/4) - V(0) > \frac{14}{13} \), and therefore, that the reporting rule helps lower the discount factor for sustaining
an efficient relational contract.

As a postscript, we note that to make the proof above self-contained, rather than calculating the explicit value of \( V(\frac{1}{4}) - V(0) \), we calculated above various lower bounds for it. Using the technique developed in our general analysis, one can show that

\[
V(0) = w^* - c + \frac{1}{13} (b^* + \delta^* V(1)) + \frac{12}{13} \delta^* V(\frac{1}{4});
\]

\[
V(1) = w^* + (b^* + \delta^* V(0)).
\]

Together with (V-A), this gives that

\[
V \left( \frac{1}{4} \right) - V(0) = \frac{3}{13} \left( 1 - \frac{3}{13} \delta^* \right) b^* - \frac{1}{13} \delta^* c = 448/183,
\]

which is greater than 14/13.

2 \( q > 0 \)

We show that spillover reporting helps the sustainability of efficient relational contracts when \( q > 0 \) by presenting two results. The first one is a continuity-type of result that shows once spillover reporting helps for \( q = 0 \), it also helps for \( q \) close to 0. For given \( c, p, \) and \( \delta \), let \( S_1 \) be the smallest expected discounted surplus for sustaining the efficient relational contract under spillover reporting when \( q = 0 \).

**Corollary 1:** Consider \( p \) and \( \delta \) such that 

\[-p/ (1 - p) + \rho^* \delta - p \left( 1 - \rho^* \right) \delta^2 > 0, \]

where \( \rho^* = p/ (p + (1 - p) \rho^*) \). For each expected discounted surplus level \( S > S_1 \), there exists an associated \( q^* > 0 \) such that efficient relational contracts can be sustained under spillover reporting for all \( q \in [0, q^*] \).

**Proof.** The proof follows from standard continuity arguments. For given \( c, p, \) and \( \delta \), let \( b^* \) be the minimum deferred bonus necessary to induce effort under spillover reporting. Proposition 1 implies that \( b^* < c/p \) when 

\[-p/ (1 - p) + \rho^* \delta - p \left( 1 - \rho^* \right) \delta^2 > 0, \]

where \( \rho^* = p/ (p + (1 - p) \rho^*) \). Now again normalize the agent’s outside option \( u \) to be 0, and for each \( w \) and \( b \), define the agent’s value functions \( V(\rho), V_s(\rho), \) and \( V_e(\rho) \) as in Proposition 1. Recall that at \( w^* \) and \( b^* \), we have \( V_e(\rho^*) = V_s(\rho^*), V_e(\rho^*) = 0, \) and \( b^* + \delta V(0) > 0 \). Since \( V_e(\rho^*) - V_s(\rho^*), V_e(\rho^*), \) and \( b + \delta V(0) \) all increase in \( b \), it...
follows that for each \( S > b^* \equiv S_0 \), there exists a small enough \( \varepsilon(S) \) such that one can find \( w \) and \( b \in (b^*, S) \) such that

\[
V_e(\rho^*) > V_s(\rho^*) + 3\varepsilon;
\]
\[
V_e(\rho^*) > 2\varepsilon;
\]
\[
b + \delta V(0) > 2\varepsilon.
\]

Next, or each \( q > 0 \), let \( V^q(\rho) \) be the agent’s value function at the beginning of a period assuming that he has accepted the principal’s offer. Let \( V^q_e(\rho) \) be the agent’s value function if he works this period and \( V^q_s(\rho) \) be the agent’s value function if he shirks. The value functions then satisfy the following:

\[
V^q(\rho) = \max\{V^q_e(\rho), V^q_s(\rho)\};
\]
\[
V^q_e(\rho) = w - c + (p + (1 - p)\rho) \max\{0, b + \delta V^q\left(\frac{p}{p + (1 - p)\rho}\right)\}
+ (1 - p - (1 - p)\rho) \max\{0, \delta V^q(\rho^*)\};
\]
\[
V^q_s(\rho) = w + (q + (1 - q)\rho) \max\{0, b + \delta V^q\left(\frac{q}{q + (1 - p)\rho}\right)\}
+ (1 - \rho)\delta V^q(\rho^*) \max\{0, \delta V^q(\rho^*)\}.
\]

It is clear that there exists a \( q^* \) such that for all \( q \in [0, q^*] \),

\[
\max\{|V^q(\rho) - V(\rho)|, |V^q_e(\rho) - V_e(\rho)|, |V^q_s(\rho) - V_s(\rho)|\} < \varepsilon \quad \text{for all } \rho \in [0, 1].
\]

As a result,

\[
V^q_e(\rho^*) - V^q_s(\rho^*)
= V^q_e(\rho^*) - V_e(\rho^*) + V_e(\rho^*) - V_s(\rho^*) + V_s(\rho^*) - V^q_s(\rho^*)
\leq -\varepsilon + 3\varepsilon - \varepsilon
= \varepsilon.
\]

Similarly,

\[
V^q_e(\rho^*) = V_e(\rho^*) + V^q_e(\rho^*) - V_e(\rho^*) > 2\varepsilon - \varepsilon = \varepsilon,
\]

and

\[
b + \delta V^q(0) = b + \delta V(0) + \delta V^q(0) - \delta V(0) > 2\varepsilon - \varepsilon = \varepsilon.
\]
These three inequalities imply that for the chosen $w$ and $b$, it is incentive compatible for the agent to accept the contract and exerts effort.

Corollary 1 states that if spillover reporting improves the sustainability of efficient relational contracts when $q = 0$, it also does so for small $q$. To see this, note that the minimum deferred bonus necessary for effort under spillover reporting when $q = 0$ is equal to $S_1$. When the expected discounted surplus $S$ is larger than $S_1$, the principal can set a deferred bonus slightly larger than $S_1$ without reneging. With the new deferred bonus, the agent strictly prefers working over shirking when $q = 0$. It follows that for small enough $q$ the agent also prefers working since his payoff from shirking is continuous in $q$. This implies that spillover reporting continues to improve the sustainability of efficient relational contracts when $q$ is small.

Beyond the continuity result above, however, it is difficult to provide general conditions for spillover reporting to improve over the full revelation of signals. The reason is that when $q > 0$, the value function $V(\rho)$ no longer has an analytical solution and it becomes difficult to check multistage deviation. As a result, our second result uses numerical methods instead. We numerically compute the minimum deferred bonus required for effort both under spillover reporting and under full revelation of signals. In particular, we compute $V(\rho)$ for each deferred-bonus level $b$ and then find the smallest deferred bonus level that sustains efficient effort.

Figure 3: Numerical Simulation
Figure 3 reports our findings for discount factor equal to 0.9 and the cost of effort equal to 2. The two panels on the left (upper left and bottom left) depict the minimum deferred bonus necessary for sustaining effort under full revelation ($b^f$). The bottom-left panel is a 3D plot that illustrates the value of $b^f$ for each $0 < q < p < 0.5$. The top-left panel is the corresponding thermograph that projects the 3D plot into a 2D graph by using colors to represent values. The colder colors reflect smaller values and the warmer colors reflect larger ones. Since $b^f$ is equal to $c/(p - q)$, the colors become colder as $p$ increases and become warmer as $q$ increases. Moreover, all $(p, q)$ pairs on the same negative 45-degree line has the same color.

Next, the two middle panels report the minimum deferred bonus for sustaining effort under spillover reporting ($b^g$), where $g$ means that the reports are garbled from the signals. The colors again become colder as $p$ increases and warmer as $q$ increases, indicating that the minimum deferred bonus under spillover reporting also decreases with $p$ and increases with $q$. Different from the two panels on the left, however, not all $(p, q)$ pairs on the same negative 45 degree line has the same color. The $(p, q)$ pairs with the same color are no longer straight lines and appear to have a slope larger than $-1$.

The two panels on the right report the differences in the minimum deferred bonuses required to sustain efficiency between the cases of full revelation and spillover reporting, namely, $b^f - b^g$. The thermograph in the upper-right panel makes it clear that there are values of $(p, q)$ such that spillover reporting lowers the minimum deferred bonus to sustain effort ($b^f - b^g > 0$). The gains from spillover reporting concentrate on the bottom left region, where the values of $p$ and $q$ are smaller. Figure 4 marks the region of $(p, g)$ in which spillover reporting lowers the minimum deferred bonus required to induce effort.
Figure 4: Region of \((p, q)\) where spillover reporting enhances efficiency

### 3 Multiple Effort Levels

Let the agent’s effort choice in period \(t\) be \(e_t \in \{0, 1, 2\}\), and the associated effort cost be given by \(c(0) = 0, c(1) = c_1 > 0\) and \(c(2) = c_2 > c_1\). The output is binary: \(Y_t \in \{0, y\}\), and

\[
\Pr\{Y_t = y\} = \begin{cases} 
  p & e_t = 2 \\
  q & e_t = 1 \\
  0 & e_t = 0 
\end{cases}
\]

where \(1 > p > q > 0\).

We assume that \(py - c_2 > qy - c_1 \geq u + v > 0\),

so the joint surplus is maximized at \(e = 2\), followed by \(e = 1\), the outside options, and \(e = 0\). Notice that we allow the relationship to be profitable even when the agent chooses \(e = 1\). This assumption is not important for the analysis but as we will see below, it allows spillover reporting to help the relationship both in the extensive and intensive margins. For expositional convenience, define the discounted expected surplus as

\[
S = \frac{\delta}{1 - \delta}(py - c_2 - u - \pi).
\]
We assume that the supervisor, and no other parties, observe the outputs. In other words, the supervisor’s signal is exactly equal to the output.

Compared to the binary-effort case with \( q > 0 \), the model above essentially adds a lower level of effort \( (e = 0) \). As a result, if one can rule out that \( e = 0 \) is used in a relational contract, the result from the binary-effort model can be directly applied. The result below shows that spillover reporting can improve the efficiency of the relational contracts when effort costs are sufficiently convex.

**Corollary 2:** For given \( c_1, c_2, p, q, \) and \( \delta \), let \( S_1 \) be the minimum expected discounted surplus for effort under spillover reporting in the binary-effort model. For any \( S \in (S_1, \frac{c_2 - c_1}{p - q}) \), efficient relational contracts are sustainable under spillover reporting when \( (c_2 - c_1)/c_1 > M \) for some \( M > 0 \).

**Proof.** Define \( V(\rho) \) as the agent’s value function at the beginning of a period assuming that he has accepted the principal’s offer. Let \( V_i(\rho) \) be the agent’s value function if he puts in effort level of \( i \in \{0, 1, 2\} \) this period. Normalize the agent’s outside option \( u \) to be 0. For each base wage \( w \) and deferred bonus \( b \), the value functions satisfy the following:

\[
V(\rho) = \max\{V_0(\rho), V_1(\rho), V_2(\rho)\}
\]

\[
V_2(\rho) = w - c_2 + (p + (1 - p)\rho) \max\{0, b + \delta V\left(\frac{p}{p + (1 - p)\rho}\right)\}
+ (1 - p - (1 - p)\rho) \max\{0, \delta V(\rho^*)\};
\]

\[
V_1(\rho) = w - c_1 + (q + (1 - q)\rho) \max\{0, b + \delta V\left(\frac{q}{q + (1 - p)\rho}\right)\}
+ (1 - q - (1 - q)\rho) \max\{0, \delta V(\rho^*)\}.
\]

\[
V_0(\rho) = w + \rho \max\{0, b + \delta V(0)\} + (1 - \rho) \max\{0, \delta V(\rho^*)\}.
\]

It is clear that for each \( w \) and \( b \), there is a unique set of value functions that satisfy the functional equations above.

Next, suppose \( b^* \) is the minimum deferred bonus for effort in the binary effort case and \( w^* \) is the associated base wage so that the agent’s participation constraint binds. Let \( V_e(\rho), V_s(\rho), \) and \( V_b(\rho) \) be the agent’s value functions associated with \( b^* \) and \( w^* \), where we use the subscript \( b \) to stand for the binary case. Now let \( w = w^* + c_1 \) and \( b = b^* \). We show below that for sufficiently small \( c_1 \), we have \( V_2(\rho) = V_e(\rho), V_1(\rho) = V_s(\rho), V(\rho) = V_b(\rho), \) and \( V_0(\rho) = w + \rho(b + \delta V_b(0)) \) for all \( \rho \).
To do this, it suffices to show that the constructed value function satisfies the set of functional equations above. Given the properties of $V_e(\rho)$, $V_s(\rho)$, and $V_b(\rho)$, we only need to show that $V_1(\rho) \geq V_0(\rho)$ for all $\rho$. Notice that

$$V_1(\rho) - V_0(\rho) = (q + (1 - q)\rho)(b + \delta V_b(q)) - \rho(b + \delta V_b(0)) - c_1$$

where the inequality uses the fact that $V$ is increasing in $\rho$. Since $b + \delta V_b(0) > 0$ (by Proposition 1) and $b = b^*$ is independent of $c_1$, it is clear that for small enough $c_1$, $V_1(\rho) - V_0(\rho) > 0$ for all $\rho$.

The above implies that for small enough $c_1$, the agent is willing to choose $e = 2$ when $b = b^* = S_0$ and $w = w^* + c_1$. Finally, since $S > S_0$, the principal will not renege on the deferred bonus. This establishes that the stationary strategies with deferred bonus with $w = w^* + c_1$ and $b = b^*$ supports the efficient relational contracts. 

Notice that $(c_2 - c_1)/c_1$ measures the convexity of effort cost since $c_2 - c_1$ is the marginal cost of effort from increasing $e = 1$ to $e = 2$ and $c_1$ is the marginal cost of effort from increasing $e = 0$ to $e = 1$. Corollary 2 therefore shows that when effort costs are sufficiently convex, spillover reporting can support the efficient relational contracts even if it is impossible to do so with perfect revelation of outputs. Notice that if the signals were revealed fully when $S \in (S_1, \frac{c_2 - c_1}{p - q})$, the agent would either choose a lower level of effort ($e = 1$) or forgo the relationship by taking his outside option. In the former case, spillover reporting improves the relationship in the intensive margin by making it more efficient. In the later case, spillover reporting improves the relationship in the extensive margin by sustaining an efficient relationship that otherwise would fail to start.

The intuition for Corollary 2 is as follows. When the cost function is sufficiently convex, the gain of deviating from $e = 1$ to $e = 0$ is small relative to the gain of deviating from $e = 2$ to $e = 1$. It follows that if the agent does not gain from deviating to $e = 1$, he will not gain from deviating to $e = 0$. Therefore, if spillover reporting helps sustain efficient relational contracts with binary effort, it can also help here when the effort cost is sufficiently convex.
This intuition suggests that spillover reporting may have wider applicability. For example, when there are multiple effort levels, ruling out profitable local deviations is sometimes enough for ruling out global deviations. Since checking local deviations requires essentially comparing payoffs from two (adjacent) effort levels, our result on spillover reporting can be applied to relax constraints that prevent local deviation. This suggests that spillover reporting can improve the sustainability of relational contracts for more general effort cost structures.

4 Multiple Agents

Suppose there are \( n \) identical agents, and the production is independent across them. For each agent \( i \), his output is \( Y_i \in \{0, y\} \). The probability that \( Y_i = y \) is \( p \) if agent \( i \) puts in effort, and is 0 otherwise. We assume that \( py - c > u + v > 0 \), where \( c \) is the agent’s cost of effort. In addition, the supervisor, and no other agents, observe the outputs. In other words, this is essentially an \( n \)-fold model of that in Section 4 with \( p = p_0 \). Below, we study two types of relationships. The first is the commonly used, but suboptimal, independent relationships. In the second one, the relationships are interdependent.

4.1 Independent Relationships

In this case, the principal deals with each of the agent separately. In other words, the relationships are independent of each other. To derive the condition for sustaining an efficient \( n \)-agent relationship with full signal revelations, note that agent \( i \) will put in effort if

\[
  b_i \geq c/p.
\]

The maximal reneging temptation of the principal occurs in the case when all agents receive a bonus. The principal’s gain from reneging is given by \( \sum_{i=1}^n b_i \).

Next, the principal’s surplus is given by

\[
  \frac{\delta n}{1 - \delta} (py - c - u - \pi) \equiv \frac{\delta ns (1)}{1 - \delta}.
\]

It follows that for the principal not to renege on the bonuses, the non-reneging con-
straint is given by
\[ \frac{\delta ns(1)}{1 - \delta} \geq \sum_{i=1}^{n} b_i. \]

Combining this with the agent’s incentive constraint, we obtain that the condition for sustaining the efficient relational contract with full signal revelation is given by
\[ \frac{\delta ns(1)}{1 - \delta} \geq \sum_{i=1}^{n} b_i \geq nc/p, \]
or equivalently,
\[ \frac{\delta ns(1)}{1 - \delta} \geq c/p. \]
This is the same condition as the single-agent case. Note that we showed above that this condition is necessary, but clearly this condition is also sufficient.

Next, to obtain the condition for sustaining an efficient \( n \)-agent relationship with spillover reporting, let \( b_g \) be the deferred bonus necessary to induce effort under spillover reporting (see the proof of Proposition 1, part (i) for the expression of \( b_g \)). Here, the subscript \( g \) reflects that the reports are garbled from the signals. This implies that each agent \( i \) puts in effort if
\[ b_i \geq b_g. \]
As before, the non-reneging constraint of the principal is given by
\[ \frac{\delta ns(1)}{1 - \delta} \geq \sum_{i=1}^{n} b_i. \]
Combining this with the agent’s incentive constraint, we obtain that the condition for sustaining the efficient (independent) \( n \)-agent relational contract with spillover reporting is
\[ \frac{\delta ns(1)}{1 - \delta} \geq \sum_{i=1}^{n} b_i \geq nb_g, \]
or equivalently,
\[ \frac{\delta s(1)}{1 - \delta} \geq b_g. \]
Again, this is the same condition as the single-agent case, and in addition, it is clear that this condition is also a sufficient one. It follows that under the independent
relationships, spillover reporting helps with the $n$-agent case if and only if it helps with the single-agent case.

### 4.2 Interdependent Relationships

When the signals are fully revealed, the independent relationships considered above are suboptimal. Levin (2002) implies that the optimal relational contract takes the following form. In each period, there’s a bonus pool of $B^p(n)$ to be shared by agents with good signals. With probability $(1 - p)^n$, however, no agent receives a good signal, and in this case, the bonus pool is not paid out.

We now derive the agent’s incentive constraint under the optimal $n$-agent relational contract (with full revelation of signals). Since the agents are symmetric under the relational contract, we suppress the subscript and let $b^p(n)$ be the expected bonus of the agent if his signal is good. The agent will put in effort if

$$b^p(n) \geq \frac{c}{p}.$$  

Notice that $b^p(n)$ and $B^p(n)$ are linked through the following equation:

$$pb^p(n) = \frac{1}{n} (1 - (1 - p)^n) B^p(n).$$

The left-hand side is the agent’s expected bonus. And the right-hand side is another way to calculate it: the expected total bonus is $(1 - (1 - p)^n) B^p(n)$, where $1 - (1 - p)^n$ is the probability that the bonus pool $B^p(n)$ is paid out, and the agent gets $1/n$ of it in expectation. The equation implies that the agent’s incentive constraint can be written as

$$B^p(n) = \frac{npb^p(n)}{1 - (1 - p)^n} \geq \frac{np}{1 - (1 - p)^n} \frac{c}{p}.$$  

Next, the principal’s non-reneging condition is given by

$$\frac{\delta ns(1)}{1 - \delta} \geq B^p(n).$$

And combining this with the agent’s incentive constraint, we obtain that the necessary condition for sustaining an efficient $n$-agent relational contract with full signal revelation is

$$\frac{\delta s(1)}{1 - \delta} \geq \frac{p}{1 - (1 - p)^n} \frac{c}{p}. \quad \text{(n-perfect)}$$
We now consider the following \( n \)-agent relational contract with spillover reporting. For each agent, the supervisor sends the report according to the spillover reporting described in Section 4. As in Section 4, the principal formally pays out no end-of-period bonus. Instead, “the deferred-bonus pool” is paid out at the beginning of the next period, and it is divided equally among the agents with good reports. Each agent observes his own report at the end of each period and also a public signal for whether at least one agent has a good report. In addition, each agent observes the contracts offered by the principal to all agents at the beginning of each period. All agents take their outside options forever if any parties ever publicly deviates.

Let \( \delta_0(n) \) be the smallest discount factor for an efficient \( n \)-agent relational contract under full revelation of signals, and \( \delta_g(n) \) the corresponding discount factor under spillover reporting. Corollary 3 below provides the condition for when spillover reporting improves over full revelation of signals.

**Corollary 3** \( \delta_g(n) < \delta_0(n) \) if and only if

\[
\frac{(p + (1 - p)\rho^*) (1 - (1 - p)^n)}{(1 - (1 - (p + (1 - p)\rho^*))^n)} \frac{1}{(p + (1 - p)\rho^*) - \rho^* \frac{1-p}{1-\delta p(1-\rho^*)}} < 1.
\]

**Proof.** At \( \delta = \delta_g(n) \), denote \( w(n) \) and \( b^g(n) \) as the agent’s base wage and expected bonus (conditional on a good report) under spillover reporting. Let \( B^g(n) \) be the corresponding deferred-bonus pool. Notice that \( b^g(n) \) and \( B^g(n) \) are linked through

\[
(p + (1 - p)\rho^*) b^g(n) = \frac{1}{n} (1 - (1 - (p + (1 - p)\rho^*))^n) B^g(n),
\]

or alternatively,

\[
B^g(n) = \frac{n (p + (1 - p)\rho^*) b^g(n)}{(1 - (1 - (p + (1 - p)\rho^*))^n)}.
\]

This implies that the principal’s non-reneging constraint can be written as

\[
\frac{\delta ns(1)}{1 - \delta} \geq B^g(n) = \frac{n (p + (1 - p)\rho^*) b^g(n)}{(1 - (1 - (p + (1 - p)\rho^*))^n)}.
\]

Next, we determine the value of \( b^g(n) \). As in Proposition 1, each agent’s expected future payoff is determined by his belief of a high signal in the previous period, and
the value functions satisfy

\[ V(\rho) = \max\{V_e(\rho), V_s(\rho)\}; \]

\[ V_e(\rho) = w(n) - c + (p + (1-p)\rho) \max\left\{0, b^g(n) + \delta V\left(\frac{p}{p + (1-p)\rho}\right)\right\} + (1-p - (1-p)\rho) \max\{0, \delta V(\rho^*)\}; \]

\[ V_s(\rho) = w(n) + \rho \max\{0, b^g(n) + \delta V(0)\} + (1-\rho) \max\{0, \delta V(\rho^*)\}. \]

Note that this is exactly the same set of functional equations as that in Proposition 1 (with \(p_0 = 0\)). As a result, we obtain that

\[ c = \left( (p + (1-p)\rho^*) - \rho^* \frac{1 - (1-p)\delta\rho^*}{1 - \delta^2 p (1-\rho^*)} \right) b^g(n). \]

Combining the above with the principal’s non-reneging constraint, we obtain that the condition for sustaining the efficient relational contract is given by

\[ \frac{\delta s(1)}{1-\delta} \geq \frac{(p + (1-p)\rho^*)}{(1 - (1 - (p + (1-p)\rho^*))^n)} \frac{c}{(p + (1-p)\rho^*) - \rho^* \frac{1 - (1-p)\delta\rho^*}{1 - \delta^2 p (1-\rho^*)}}. \]  

Comparing this condition to the condition for full revelation of signals, we get that spillover reporting is an improvement if and only if

\[ \frac{(p + (1-p)\rho^*)}{(1 - (1 - (p + (1-p)\rho^*))^n)} \frac{c}{(p + (1-p)\rho^*) - \rho^* \frac{1 - (1-p)\delta\rho^*}{1 - \delta^2 p (1-\rho^*)}} \leq \frac{p}{1 - (1-p)^n} \frac{c}{p}. \]  

This finishes the proof. □

Notice that when \(n = 1\), this condition in Corollary 3 is equivalent to that in part (i) of Proposition 1 (with \(p_0 = 0\)). As \(n\) increases, it can be checked that the left-hand side increases with \(n\), so this condition becomes more difficult to satisfy. It can be shown, however, that for all \(n\), there exists ranges of \(p\) such that spillover reporting helps to sustain relationship. Moreover, for all \(n\), the minimal surplus for an efficient relational contract under full revelation is about \(\delta\) times more as that under spillover reporting when as \(p\) goes to 0.

**Corollary 4** Let \(S_0(p, \delta, n)\) be the minimal surplus necessary sustain an efficient
relational contract with full revelation of signal. Let \( S_g(p, \delta, n) \) be the corresponding surplus with spillover reporting. For all \( n \),

\[
\lim_{p \to 0} \frac{S_0(p, \delta, n)}{S_g(p, \delta, n)} = 1 + \delta.
\]

**Proof.** From the condition on the sustainability of efficient relational contracts, we have

\[
\frac{S_0(p, \delta, n)}{S_g(p, \delta, n)} = \frac{(p + (1 - p) \rho^*) - \rho^* \frac{1-(1-p)\delta \rho^*}{1-\delta^2 p(1-\rho^*)}}{p} \frac{A(p, n)}{B(p, n)},
\]

where \( A(p, n) = \frac{p}{1-(1-p)\rho^*} \) and \( B(p, n) = \frac{(p+(1-p)\rho^*)}{(1-(1-(1-p)p\rho^*))} \), and recall that \( \rho^* = \frac{p}{p + (1-p) \rho^*} \).

Now notice that we have \( p = \frac{(\rho^*)^2}{1-\rho^*(\rho^*)^2} \), so \( \lim_{p \to 0} \rho^* = 0 \), and \( \lim_{p \to 0} \frac{(\rho^*)^2}{p} = 1 \). Now

\[
\lim_{p \to 0} \frac{S_0(p, \delta, n)}{S_g(p, \delta, n)} = \lim_{p \to 0} \frac{(p + (1 - p) \rho^*) - \rho^* \frac{1-(1-p)\delta \rho^*}{1-\delta^2 p(1-\rho^*)}}{p} \lim_{p \to 0} \frac{A(p, n)}{B(p, n)}.
\]

And for each \( n \), \( \lim_{x \to 0} \frac{x}{1-(1-x)^n} = \frac{1}{n} \), so \( \lim_{p \to 0} A(p, n) = \lim_{p \to 0} B(p, n) = \frac{1}{n} \). In addition,

\[
\lim_{p \to 0} \frac{(p + (1 - p) \rho^*) - \rho^* \frac{1-(1-p)\delta \rho^*}{1-\delta^2 p(1-\rho^*)}}{p} = 1 + \lim_{p \to 0} \frac{\rho^* - p + (1 - p) \delta (\rho^* - \delta p (1 - \rho^*))}{p(1 - \delta^2 p(1 - \rho^*))}
\]

\[
= 1 + \frac{\rho^* - p + (1 - p) \delta (\rho^* - \delta p (1 - \rho^*))}{p(1 - \delta^2 p(1 - \rho^*))}
\]

Together, we then have

\[
\lim_{p \to 0} \frac{S_0(p, \delta, n)}{S_g(p, \delta, n)} = 1 + \delta.
\]
5 Collusion

Suppose the principal can offer the supervisor the following history-dependent contract at the beginning of each period $t$,

$$w_s^t + \alpha_t Y_0,$$

where $\alpha$ is the share of the public output that goes to the supervisor. To simplify the analysis, we assume that $w_s^t = -\alpha_t p_0 y_0$ so that the supervisor receives 0 expected payoff in equilibrium. We also assume that $\alpha$ share of the output directly accrues to the supervisor, and this allows us to focus on the relational contract between the principal and the agent. Now consider the following type of stationary collusion. The agent offers the supervisor a payment of $t_b > 0$ in each period. In return, the supervisor always sends a good report about the agent.

Now define $p_g \equiv p_s + (1 - p_s) \rho^*$ be the probability that the agent receives a deferred bonus under spillover reporting (when he works and does not engage in stationary collusion). Our result below shows that when $\delta < 1/(2 - p_g)$, if spillover reporting can help sustain the efficient relational contract, then the principal can prevent stationary collusion by giving enough share of the output to the supervisor.

**Corollary 5:** Suppose spillover reporting helps sustain the relational contract, i.e., $1 - p_0 - \delta \rho^* > 0$ and $-p_s/(1 - p_s) + \delta (\rho^* - \delta (1 - \rho^*)) > 0$. Now if $\delta < 1/(2 - p_g)$, then there exists $\alpha^* < 1$ such that for all $\alpha > \alpha^*$, the supervisor and the agent will not engage in stationary collusion.

**Proof.** Let $b_g$ be the lowest deferred bonus necessary to sustain effort under spillover reporting. From the proof of Proposition 1, we have

$$b_g = \frac{c}{p_0 + (1 - p_0)(p + (1 - p)\rho^*) - \rho^* \frac{1 - (1-p_0)(1-p)\delta\rho^*}{1 - \delta^2 p(1-\rho^*)}}.$$

To prevent stationary collusion, it suffices to show that the joint payoff between the supervisor and the agent without collusion is smaller than their joint payoff under collusion. The joint payoff without collusion is 0 by design.

Now if the agent colludes with the supervisor, he will choose to shirk. This is because if the agent always receives a good report, he always receives a deferred bonus, and therefore, does not have an incentive to put in effort. Given the agent
shirks, the supervisor’s payoff is given by $t_b - \alpha p_0 y_0$ per period. For the agent, his payoff under collusion is given by

$$-t_b + c + (1 - p_g) b_g,$$

where recall that $p_g = p_s + (1 - p_s) \rho^*$ is the probability that a deferred bonus is paid out to the agent under spillover reporting when the agent works.

It follows that the joint payoff of the agent and the supervisor is given by $-\alpha p_0 y_0 + c + (1 - p_g) b_g$. As a result, they will not collude if

$$\alpha p_0 y_0 - c \geq (1 - p_g) b_g.$$  \hfill (No-collusion condition)

It follows that a large enough $\alpha$ can be chosen as long as

$$s(1) = p_0 y_0 - c \geq (1 - p_g) b_g.$$  

To see that this condition is satisfied when $\delta < 1/(2 - p_g)$, recall that for a relational contract to be sustainable under spillover reporting, the principal’s non-reneging condition is given by

$$\frac{\delta}{1 - \delta} s(1) \geq b_g.$$  

It follows that

$$s(1) \geq \frac{1 - \delta}{\delta} b_g > (1 - p_g) b_g,$$

where the last inequality follows because $\delta < 1/(2 - p_g)$.

The main idea for why collusion will not take place is as follows. If the collusion occurs so that the supervisor always sends out good reports, the agent will shirk. This will then hurt the supervisor’s payoff, and the damage is larger when the supervisor receives a bigger share of the output. For a large enough $\alpha$, the cost from agent shirking is sufficiently large that the supervisor and the agent will not engage in collusion.

6 General Reporting Rules

Below, we show that any reporting rule with a predetermined restart date cannot help sustain an efficient relational contract. Proposition 2 in the main text follows
directly from it. We also provide a proof for Proposition 3.

Proposition 2': Let \((U_t, \Pi_t)\) be the expected discounted payoffs of the agent and the principal evaluated at time \(t\). Suppose \((??)\) fails. For all \(t\), if there exists a predetermined \(t' \geq t\) such that \((U_{t'}, \Pi_{t'})\) are independent of \(h_{t'}\), then \(e_t = 0\).

Proof. Note that in our setting, the feasible surplus of the game \(S\) cannot be raised by any reporting rule. Suppose \((U_{t'}, \Pi_{t'})\) are independent of \(h_{t'}\). Then \(e_{t'-1}\) is solely motivated by \(b_{t'-1}\), and since \(b_{t'-1} \leq S < c/(p - q)\), it must hold that \(e_{t'-1} = 0\). Given that \(e_{t'-1} = 0\) regardless of \(h_{t'}\) and that \((??)\) fails, the expected sum of bonuses \(E(b_{t'-2} + \delta b_{t'-1})\) should also be no larger than \(S\). This implies that \(e_{t'-2} = 0\) as well. By induction, \(e_{\tau} = 0\) for all \(\tau \in \{1, 2, ..., t'\}\). ■

Proposition 3: Let \(s(1)\) be the expected future surplus of the relationship per period. For any reporting rule to sustain the efficient relational contract, one must have

\[
\frac{\delta s(1)}{1 - \delta} \geq \sqrt{4p(1 - p)} \frac{c}{p}.
\]

In particular, full revelation of signals is the optimal reporting rule when \(p = 1/2\).

Proof. Define \(S = \delta s(1)/(1 - \delta)\). When the signals are perfectly revealed, the necessary and sufficient condition for sustaining an efficient relational contract is given by

\[
\frac{c}{p} \leq S.
\]

We want to show that if \(S < \sqrt{4p(1 - p)} \frac{c}{p}\), it is impossible to construct an equilibrium in which the agent puts in effort. In particular, using standard argument as in Fuchs (2007), it suffices to show that there does not exist an equilibrium in which the agent always puts in effort (unless the relationship is terminated).

Consider an arbitrary information partition process. Pick one information set \((h^t)\). Use \(x\) to denote the possible states within the information set. One interpretation of \(x\) is some output realizations \(y^t\) that falls into \(h^t\).

Let \(V(x)\) be the agent’s continuation payoff in state \(x\) after \(e_{t+1}\) is put in but before \(y_{t+1}\) is realized and \(W_{t+1}\) is paid out. Let \(V(x_i)\) be the agent’s continuation payoff in state \(x\) after \(e_{t+1}\) is put in, \(y_{t+1}\) is realized but before \(W_{t+1}\) is paid out. Within each state \(x\), we have \(x_i \in \{x_y, x_0\}\), where \(x_y\) denotes that \(Y_t = y\) is realized following \(x\), and \(x_0\) denotes that \(Y_t = 0\) is realized.

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Note that 
\[ V(x) = V(x) + p(V(x_y) - V(x)) + (1 - p)(V(x_0) - V(x)). \]

And since the output \( Y_t \) is independent of the past state, we have \( \text{Cov}(V(x_i) - V(x), V(x)) = 0. \)

To induce effort, we need 
\[ E_x[V(x_y) - V(x_0)] = E_x[V(x_y) - V(x)] - E_x[V(x_0) - V(x)] \geq \frac{c}{p}. \]

This helps give a lower bound for \( \text{Var}(V(x_i)) \). In particular,

\[
\text{Var}(V(x_i)) = \text{Var}(V(x)) + \text{Var}(V(x_i) - V(x)) \\
= \text{Var}(V(x)) + E_y[\text{Var}(V(x_i) - V(x))|Y] \\
+ \text{Var}(E_x[V(x_i) - V(x)]|Y) \\
\geq \text{Var}(V(x)) + E_y[\text{Var}(V(x_i) - V(x))|Y] \\
+ p(1 - p)(\frac{c}{p})^2 \\
\geq \text{Var}(V(x)) + p(1 - p)(\frac{c}{p})^2,
\]

where the first line follows because \( \text{Cov}(V(x_i) - V(x), V(x)) = 0 \), the second line uses the variance decomposition formula, the third line follows because \( E_x[V(x_i) - V(x)]|Y \) is a binary value \( (Y \in \{0, y\}) \) such that with probability \( p \) its value is \( E_x[V(x_y) - V(x)] \) and with probability \( 1 - p \) its value is \( E_x[V(x_0) - V(x)] \), and \( E_x[V(x_y) - V(x)] - E_x[V(x_0) - V(x)] \geq c/p. \)

Now let’s provide an upper bound for \( \text{Var}(V(x_i)) \). Suppose a public report \( s(x_i) \) will be sent out after state \( x_i \). Let \( b(s) \) be the bonus paid out to the agent (at the end of the period) following report \( s \). This allows us to write

\[ V(x_i) = b(s(x_i)) + \delta V_s(x_i)(x_i), \]

where \( V_s(x_i)(x_i) \) is the continuation payoff of \( x_i \), which goes to the information set by report \( s(x_i) \).

Note that for the principal to be willing to pay the bonus, we must have

\[
\max_s \{b_s + \delta E_{x_i}[V_s(x_i)|s]\} - \min_s \{b_s + \delta E_{x_i}[V_s(x_i)|s]\} \leq S.
\]
Because otherwise the expected payoff of the principal following some report will be below his outside option.

Decomposing the variance on the reports, we have

\[
Var(V(x_i)) = Var(E[b_s + \delta V_s(x)|s]) + E[Var(b_s + \delta V_s(x)|s)] \leq \frac{1}{4}S^2 + \delta^2 E[Var(V_s(x)|s)].
\]

Now combining the upper and lower bound for \(Var(V(x_i))\), we get that

\[
\frac{1}{4}S^2 + \delta^2 E[Var(V_s(x_i)|s)] \geq Var(V(x)) + p(1 - p)(\frac{c}{p})^2,
\]

or equivalently,

\[
E[Var(V_s(x_i)|s)] \geq \frac{1}{\delta^2} (Var(V(x)) + p(1 - p)(\frac{c}{p})^2 - \frac{1}{4}S^2).
\]

Now if \(4p(1 - p)(\frac{c}{p})^2 > S^2\), the inequality above implies that

\[
E[Var(V_s(x_i)|s)] > \frac{1}{\delta^2} (Var(V(x))).
\]

In particular, there will be one information set (associated with a signal) whose variance exceeds \(\frac{1}{\delta^2} (Var(V(x)))\). Now we can perform the same argument on this new information set, and we can construct a sequence of information set whose variance approaches infinity. This leads to a contradiction.

Therefore, to sustain an efficient relational contract, one must have

\[
S^2 \geq 4p(1 - p)(\frac{c}{p})^2.
\]

It follows that when \(p = 1/2\), this condition becomes \(S \geq \frac{c}{p}\), which is exactly the condition for sustaining the efficient relational contract under full revelation of signals. This shows that full revelation of signals is the optimal reporting strategy when \(p = 1/2\).