Relational contracts, limited liability, and employment dynamics

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Abstract

This paper studies a relational contracting model in which the agent is protected by a limited liability constraint. The agent’s effort is his private information and affects output stochastically. We characterize the optimal relational contract and compare the dynamics of the relationship with that under the optimal long-term contract. Under the optimal relational contract, the relationship is less likely to survive, and the surviving relationship is less efficient. In addition, relationships always converge to a steady state under the optimal long-term contract, but they can cycle among different phases under the optimal relational contract. © 2017 Elsevier Inc. All rights reserved.

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1. Introduction

Many business relationships have three salient features. First, the relationships are ongoing, and the business parties interact repeatedly. Second, some party in the relationship has a moral hazard problem, and can take an action that is privately observed. Third, the party with the moral hazard problem also has limited liability, and he can only be punished to a degree. Examples with these features include banks lending to entrepreneurs, manufacturers outsourcing to suppliers, and firms hiring workers. In these examples, the relationships are typically repeated; the entrepreneurs, suppliers, and workers often take private actions to advance their own interests; they are protected by limited liability: entrepreneurs and suppliers can declare bankruptcy, and workers are protected by minimum wages.

The literature on dynamic contracting with limited liability has captured these features, and has been successful in providing insights into the dynamics of the relationship, with applications, for example, in corporate finance; see Sannikov (2013a) for a survey. A key assumption in this literature is that there are formal long-term agreements for the relationships, and specifically, the principal can commit to these agreements.

However, in many situations—employment relationships in particular, the costs of drafting and enforcing long-term agreements can be prohibitively high. Consequently, the principal cannot be expected to commit to formal long-term agreements. The relationships depend, instead, on relational contracts in which the parties keep their agreement because of their concerns for their future loss. The main purpose of the paper is to investigate how the lack of commitment affects the dynamics of the relationship.

Specifically, we study a model of relational contracts with imperfect monitoring—an infinitely repeated principal–agent model where output is publicly observable but not contractible.1 The agent privately chooses to work or shirk, and by working the agent increases the probability of high output. The production environment is similar to models of financial contracting—DeMarzo and Fishman (2007), Biais et al. (2004), Biais et al. (2013) (hereafter Biais et al. (2004, 2013)), but importantly, the principal cannot commit to long-term contracts. We model the limited liability constraint by requiring that the agent’s pay each period does not fall below an exogenously given wage floor $w$. Following Levin (2003), we define a relational contract as a Perfect Public Equilibrium (PPE) of the game. The optimal relational contract is the PPE that maximizes the principal’s payoff.

We characterize the set of PPE payoffs and derive properties of the optimal relational contract. The PPE payoff frontier consists of a punishment region in the left, a reward region in the right, and a probationary region in the middle. The agent’s payoff in the optimal relational contract starts in the probationary region, where the agent puts in effort and receives wages equal to $w$. If the output history has been sufficiently favorable, the agent’s continuation payoff moves to the reward region, which pays wages above $w$. If the output history has been sufficiently unfavorable, the agent’s continuation payoff moves to the punishment region, where the agent is either asked to shirk for some periods of time, or the relationship is terminated. The three regions of the PPE payoff frontier—and the structure of the optimal relational contract they induce—reflect the general lesson that rewards (and punishments) should be backloaded in repeated interactions, and are also featured in dynamic contracting models.

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1 For a definitive treatment of relational contracts with observable actions, see MacLeod and Malcomson (1988).
There are, however, important differences when the principal cannot commit. First, whereas the reward region is absorbing with commitment, it can be non-absorbing—when the discount factor is low—without commitment. When the principal cannot commit, the amount of the surplus in the relationship constrains the reward she can give out. As a result, the back-loading of the reward is incomplete when the surplus is low so that, once output is low, the agent’s payoff moves out of the reward region. Second, lack of commitment changes the way the agent is punished. For some parameter values, the agent is punished via shirking under the optimal long-term contract, but via termination under the optimal relational contract. This occurs because lack of commitment reduces the value of the relationship, making termination a less costly way of punishment.

These differences imply that lack of commitment has a number of implications on the survival and wage pattern of a relationship. When the relationship does not have a high enough surplus or when players are not too patient, non-commitment makes the relationship less likely to survive. For some parameter ranges, the relationship survives with probability 1 under the optimal long-term contract, but with probability 0 under the optimal relational contract. Even if the relationship survives in the long run, non-commitment reduces its efficiency, lowers the agent’s wage, and flattens the agent’s wage-tenure profile. In addition, whereas the relationship always converges to a steady state in the long run under long-term contracts, it can cycle among different phases under the relational contract.

This paper contributes to the literature of dynamic principal–agent models without commitment; see for example, Malcomson (2013) for a survey. One closely related paper is Hörner and Samuelson (2016), who also study a discrete-time model of repeated moral hazard without commitment. In each period, a principal finances a project, chooses its scale, and pays an agent according to his report. The agent privately observes the outcome of the project and can appropriate the return of a successful project by reporting it fails. Unlike us, Hörner and Samuelson (2016) assume that the principal can commit to the contingent payments within each period, so that she can use payments alone—and not the future scales of the project—to induce the agent to report truthfully. This implies that the project’s future scales are always efficient once it reaches its efficient scale. In addition, because the scale of the project is continuous, when the agent reports failure in one period, rather than having a probabilistic termination (as can happen in our model), the scale of the project next period decreases deterministically and never reaches zero, implying that the relationship never terminates.

Another related literature is dynamic principal–agent models with commitment—particularly models of dynamic contracting; see Biais et al. (2013) and Sannikov (2013b) for surveys. The two most related papers are Biais et al. (2004) and Biais et al. (2013). Both include analyses of discrete-time models in which the agent is risk-neutral, has a limited liability constraint, and has an equal discount factor as the principal. In these papers, because the principal can commit, the

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2 Other recent papers in this literature include Fuchs (2007), Halac (2012), Padro i Miguel and Yared (2012), Li and Matouschek (2013), and Yang (2013). There are also a number of models without commitment but with perfect information; see for example, Thomas and Worrall (1994), Ray (2002), Albuquerque and Hopenhayn (2004), and Thomas and Worrall (2010). Because information is perfect, the relationships become more efficient over time in these models.

3 See, Green (1987), Spear and Srivastava (1987), and Thomas and Worrall (1990) for earlier contributions to this literature. In addition, many papers in this literature assume that the agent has a limited liability constraint that is weakly greater than the agent’s per period outside option. With this assumption, there is often no distinction whether the agent can or cannot quit the relationship; see for example, DeMarzo and Sannikov (2006) for a discussion of this implication.
reward region is absorbing. In contrast, the lack of commitment in our model implies that the reward region is non-absorbing when the relationship has low surplus.

Two other closely related papers with full commitment are DeMarzo and Fishman (2007) and Zhu (2013). DeMarzo and Fishman (2007) consider a similar, but more general, principal–agent model in discrete-time, but they focus on the case in which the time horizon is finite, and not on the long-run properties of the model. Zhu (2013) considers a continuous-time model, and as in our paper, allows the principal to punish the agent with shirking in addition to termination. In Zhu (2013), the principal is more patient than the agent, so that the reward region is non-absorbing, and in the long run, the relationship either survives with probability 0 (when termination is used) or probability 1 (when shirking is used). In contrast, the reward region is absorbing in our model when the relationship has high surplus. In this case, the relationship survives with a positive—but less than 1—probability.

The rest of the paper is organized as follows. We set up the model in Section 2. In Section 3, we carry out an preliminary analysis and describe the optimal long-term contract benchmark. In Section 4, we characterize the optimal relational contract and explore its implications. Section 5 concludes. All proofs are relegated to the Appendix A.

2. Setup

There is one principal and one agent. Both are risk neutral, infinitely lived, and have a common discount factor $\delta$. Time is discrete and indexed by $t \in \{1, 2, \ldots, \infty\}$. At the beginning of each period $t$, the principal decides whether to offer a contract to the agent: $d_t^p \in \{0, 1\}$. If a contract is offered, it specifies an enforceable wage $w_t \geq \underline{w}$, where $\underline{w} \in R$ is a given wage floor.\footnote{In Zhu (2013) and several other continuous-time repeated principal–agent models (DeMarzo and Sannikov (2006), Biais et al. (2007)), the output-generating process is continuous. In our setting, a continuous process is ill-suited for capturing the effect of non-commitment because in its limiting discrete-time analog, as the time interval goes to 0 so goes the per-period reward, making the principal essentially able to commit. To capture the effect of non-commitment, a jump-process (such as Poisson) is more appropriate because it allows the rewards—whenever paid out—to stay bounded away from 0 in the continuous-time limit.}

In many relational contracting models, the contract also includes a discretionary end-of-the-period bonus. The current setup is chosen to simplify notation and to facilitate the comparison with efficiency wage models. These two setups are equivalent: they have the same set of equilibrium payoffs, and the results here can be directly translated into one with bonus.\footnote{Notice that the limited-liability constraint is not a one-off constraint, but is required to hold period-by-period.} Specifically, for any wage $w_{t+1} > \underline{w}$, principal can equivalently pay a bonus equal to $(w_{t+1} - \underline{w})/\delta$ at the end of period $t$ and a wage $\underline{w}$ at the beginning of period $t + 1$. In our discussion below, we sometimes refer to $(w_{t+1} - \underline{w})/\delta$ as the “bonus payment”.

If the principal offers a contract, the agent decides whether to accept or reject it: $d_t^A \in \{0, 1\}$. If he accepts, the principal pays wage $w_t$, the agent chooses effort $e_t \in \{0, 1\}$, and output $Y_t \in \{0, y\}$ is realized. If the agent works ($e_t = 1$), he incurs a cost of effort $c$, and $Y_t$ is equal to $y$ with probability $p \in (0, 1)$ and 0 with probability $1 - p$. If he shirks ($e_t = 0$), no effort cost is incurred, and $Y_t$ is equal to $y$ with probability $q < p$. The agent’s effort choice is his private information. Output is publicly observed.

If the principal does not offer a contract ($d_t^P = 0$) or if the agent rejects it ($d_t^A = 0$), the players receive their outside options for the period. The agent’s per-period outside option is $u$.\footnote{MacLeod and Malcomson (1998) provide a formal proof for a model with symmetric information. Their proof can be adapted to our model.}
and the principal’s is \( v \). At the end of the period, both the principal and the agent observe the realization of a random variable \( x_t \) that is uniformly distributed between 0 and 1, and we also assume that a random variable \( x_0 \) is realized at the very beginning of the game.\(^7\) To make the analysis interesting, we assume that the value of the relationship exceeds the sum of outside options if and only if the agent puts in effort: \( py - c > u + v > qy \). We summarize the timing of the stage-game in Fig. 1.

The stage-game is repeated infinitely. At the beginning of any period \( t \), the expected payoffs to the principal and the agent are given by

\[
v_t = (1 - \delta) \left[ \sum_{\tau = t}^{\infty} \delta^{t-\tau} [d^P_t d^A_t (y(q + (p - q)e_\tau) - w_\tau) + (1 - d^P_t d^A_t) v] \right];
\]

\[
u_t = (1 - \delta) \left[ \sum_{\tau = t}^{\infty} \delta^{t-\tau} [d^P_t d^A_t (w_\tau - ce_\tau) + (1 - d^P_t d^A_t) u] \right],
\]

where we multiply throughout by \((1 - \delta)\) to express the payoffs as per-period averages.\(^8\)

To describe the strategies formally, denote \( h_t = \{d^P_t, w_t, d^A_t, y_t, x_t\} \) as the public events that occur in period \( t \). Let \( h^t = \{h_n\}_{n=t}^{t-1} \) be a public history path at the beginning of period \( t \), and \( h^1 = \{x_0\} \). Let \( H^t = \{h^t\} \) be the set of public history paths until time \( t \), and define \( H = \cup_t H^t \) as the set of public histories. In period \( t \), depending on the public history \( h^t \), the principal decides whether to offer a contract to the agent \( (d^P_t) \), and if so, what wage to offer \( (w_t) \). The agent then decides whether to accept the principal’s contract \( (d^A_t) \), and if he does, what effort level to choose \( (e_t) \). Denote the principal’s public strategy—the sequence of mappings from the set of public histories to the set of feasible actions—by \( s^P \) and the agent’s public strategy by \( s^A \).\(^9\) Denote \( S^P \) and \( S^A \) as the principal’s and the agent’s set of public strategies respectively. Let \( v(s^P, s^A | h^t) \) and \( u(s^P, s^A | h^t) \) be the principal’s and the agent’s expected payoffs following public history \( h^t \).

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\(^7\) The public randomization device is a commonly made assumption in models of repeated games to convexify the equilibrium payoffs; see, for example, Mailath and Samuelson (2006, Section 3.4) for a discussion of its roles.

\(^8\) Note that it is without loss of generality to have deterministic \( d^P_t \) and \( d^A_t \) since a randomization device is available. Specifically, \( d^P_t \) and \( d^A_t \) affect \( v_t \) and \( u_t \) only through the probability of continuing the relationship \( (d^A_t d^P_t) \), which can be obtained using \( x_{t-1} \), the public randomization that occurs at the end of period \( t - 1 \).

\(^9\) The restriction to the use of public strategy is without loss of generality: the principal’s actions are public information, so our game has a “product structure.” In this case, the agent does not gain by conditioning his actions on past private effort choices; see Fudenberg and Levine (1994).
The solution concept we use is perfect public equilibrium (PPE). A PPE is a strategy profile such that a) players choose actions that depend on the public history, and b) the strategy profiles following any public history form a Nash Equilibrium of the game following that history. A strategy profile \((s^P, s^A)\) is a PPE when for each public history \(h^t\),

\[
\begin{align*}
    s^P &\in \arg\max_{s^P \in S^P} v(s^P, s^A | h^t), \\
    s^A &\in \arg\max_{s^A \in S^A} u(s^P, s^A | h^t).
\end{align*}
\]

A relational contract as a PPE of the repeated game. The optimal relational contract is the PPE that maximizes the principal’s payoff at the beginning of the game.

We solve for the optimal relational contract in the next section. To make the analysis interesting, we assume the following. First, the wage floor is sufficiently high so that the limited-liability condition matters:

\[
w > u - \frac{qc}{p-q} \equiv \frac{\bar{w}}{c}.
\]

Notice that \(w - c + pc/(p-q) = u\), so \(w\) is the cutoff wage floor in a one-period model at which the principal can motivate the agent—by paying a wage \(w\) and a bonus \(c/(p-q)\)—without giving him any rent. When \((\text{LLB})\) fails, the optimal relational contract is stationary: the principal sets a base wage equal to \(w\) and offers a discretionary “bonus payment” of \(c/(p-q)\).

Second, a non-trivial relational contract exists:

\[
p y - c - u - v > \max \{w - c - u, \frac{1 - p\delta}{\delta} \frac{c}{p-q}\}.
\]

This condition ensures the existence of the following efficiency-wage type relational contract: the agent puts in effort and gets a fixed wage \(\tilde{w} \equiv \max \{w, u + (1 - \delta q) c/\delta (p-q)\}\) in each period if the relationship continues, and the relationship terminates whenever output is low. Here, \(\tilde{w}\) is the lowest feasible effort-inducing wage since when the agent is paid \(u + (1 - \delta q) c/\delta (p-q)\), his future (normalized) rent in the relationship is \((1 - \delta) c/\delta (p-q)\)—the lowest rent to induce effort. When the agent is paid \(\tilde{w}\), \((\text{NT})\) ensures that the principal’s payoff exceeds \(u\), and therefore making the strategy a PPE. The first inequality in \((\text{NT})\) \((py - c - u - v > w - c - u)\) is equivalent to \(py - w \geq y\), so that when \(\tilde{w} = w\), the principal’s payoff exceeds \(y\). The second inequality is the same as \(((1 - \delta)/(1 - p\delta)) (py - c - u - v) > (1 - \delta) c/(\delta (p-q))\), where the left hand side is the relationship’s normalized total surplus (since the relationship ends with probability \(p\) each period), and the right hand side is the agent’s normalized rent when \(\tilde{w} > w\). Since the relationship’s surplus is greater than the agent’s rent, the principal’s payoff is bigger than \(y\).

3. Preliminary analysis

3.1. Recursive representation

To find the optimal relational contract, it is standard to first characterize the PPE payoff set, using the recursive technique developed by Abreu et al. (1990). The subsection provides a recursive representation of the PPE payoff set, which we denote by \(\mathcal{E}\), by establishing the conditions each PPE payoff must satisfy.
For each payoff in $\mathcal{E}$, the recursive representation consists of an action profile in the current period and continuation payoffs in $\mathcal{E}$ that depend on publicly observable outcomes. It is without loss of generality to assume that if a publicly observable deviation occurs, the parties terminate their relationship, as this gives all parties their respective minmax payoffs. Consequently, only the description of actions and continuation payoffs along the equilibrium path is needed.

Consider a PPE payoff pair $(u, v) \in \mathcal{E}$. The possible action profiles that support $(u, v)$ include effort, shirking, and no-entry ($d^A d^P = 0$). For $(u, v)$ to be supported with effort, the equilibrium specifies a wage payment $w$, the agent’s and the principal’s continuation payoffs following a low output $(u_l, v_l)$, and those following a high output $(u_h, v_h)$. In addition, the following conditions must hold: first, the wages satisfy the limited-liability condition:

$$w \geq w^*;$$

(LLL)

second, it is incentive compatible for the agent to exert effort:

$$\delta (p - q) (u_h - u_l) \geq (1 - \delta) c;$$

(INFO)

third, both parties’ payoffs equal the weighted sum of current and future payoffs:

$$v = (1 - \delta) (py - w) + \delta (pv_h + (1 - p) v_l),$$

(PKP)

$$u = (1 - \delta) (w - c) + \delta (pu_h + (1 - p) u_l);$$

(PKA)

and, finally, the continuation payoffs are in the set of PPE payoffs:

$$(u_l, v_l) \in \mathcal{E},$$

(SEC)

$$(u_h, v_h) \in \mathcal{E}.$$  

(SEH)

Note that constraints (SEC) and (SEH) contain the standard non-reneging constraints in models with bonus payments. To see this, consider, for example, instead of having a continuation payoff of $v_h$ next period, the principal has a continuation payoff $\hat{v}_h = v_h + (1 - \delta) \hat{b}_h/\delta$ and pays a bonus $\hat{b}_h$ at the end of this period (so as to preserve (PKP)). It is incentive compatible for the principal to pay the bonus if it is smaller than her future surplus in the relationship, i.e., $(1 - \delta) \hat{b}_h \leq \delta (\hat{v}_h - \delta \hat{v})$, which is the familiar non-reneging constraint. This constraint is the same as $v_h \geq v$, which is implied by $(u_h, v_h) \in \mathcal{E}$ because any PPE payoff of the principal must give her at least $v$, her minmax payoff. Also note that if the principal can commit to a long-term contract, the non-reneging constraint is no longer relevant. As a result, the principal’s continuation payoff can fall below his outside option in a long-term contract, but not under a relational contract.

Next, suppose $(u, v)$ is supported with shirking. The equilibrium specifies a wage payment $w$ and continuation payoff $(u_s, v_s)$, where the subscript $s$ denotes that the agent is asked to shirk this period. Because the agent is asked not to work, his incentive constraint (IC) is irrelevant. The promise-keeping constraints are given by

$$u = (1 - \delta) w + \delta u_s,$n

$$v = (1 - \delta) (qy - w) + \delta v_s,$n

and the self-enforcing constraint requires $(u_s, v_s) \in \mathcal{E}$.

Third, suppose $(u, v)$ is supported with no-entry. The parties take their outside options for one period, and their continuation payoffs are given by $(u_x, v_x)$. To support $(u, v)$ with no-entry, (LL) and (IC) are irrelevant. The promise-keeping constraints give
\[ u = (1 - \delta) u + \delta u_x, \]
\[ v = (1 - \delta) v + \delta v_x, \]
and the self-enforcing constraints require \((u_x, v_x) \in \mathcal{E}\).

Finally, \((u, v)\) can be supported with randomization, so it is a convex combination of payoffs in \(\mathcal{E}\), using the public randomization device.

To characterize \(\mathcal{E}\), we need to determine, for each \((u, v) \in \mathcal{E}\), whether it is supported by effort, shirking, no-entry, or randomization. We then need to specify the corresponding actions and continuation payoffs, and in the case of randomization, how \((u, v)\) is randomized.

### 3.2. Optimal long-term contract benchmark

To better illustrate the role of (lack of) commitment, we first describe the optimal long-term contract benchmark. When the principal can commit to a long-term contract, our setup is closely related to the framework of Biais et al. (2004, 2013), and the optimal long-term contract has features similar to theirs—with some differences to be discussed later. To facilitate the comparison, we set \(w = u = 0\) in this subsection. Denote \(u\) as the agent’s normalized payoff, and let \(f_{LR}(u)\) be the principal’s highest payoff under any long-term contract. The principal’s payoff under the optimal long-term contract is then given by \(\max_{u \in [0, \infty)} f_{LR}(u)\). The proposition below describes \(f_{LR}\) and the optimal long-term contract.

**Proposition 0.** Suppose \(w = u = 0\). There exist \(u^{*}_{LT, 1} = (1 - \delta) q c / (p - q)\) and \(u^{*}_{LT, 2} = q c / (p - q)\) that the following holds.

(i) (Punishment Region) For all \(u \in [0, u^{*}_{LT, 1}]\), \(f_{LR}(u) = v + f_{LR}(u^{*}_{LT, 1}) u / u^{*}_{LT, 1}\). \(f_{LR}(u)\) can be supported with randomization between \((0, v)\) and \((u^{*}_{LR, 1}, f_{LR}(u^{*}_{LR, 1}))\). Termination occurs when \(u = 0\).

(ii) (Probationary Region) For all \(u \in [u^{*}_{LT, 1}, u^{*}_{LT, 2}]\), \(f_{LR}(u) = (1 - \delta) p y + \delta [p f_{LR}(u_{h}(u)) + (1 - p) f_{LR}(u_{l}(u))]\). The agent puts in effort and receives \(w(u) = 0\). His continuation payoff moves to \(u_{l}(u) = (u - (1 - \delta) q c / (p - q)) / \delta\) when output is low, and to \(u_{h}(u) = u_{l}(u) + k\) when output is high, where \(k = (1 - \delta) c / (\delta (p - q))\).

(iii) (Reward Region) For \(u > u^{*}_{LT, 2}\), \(f_{LR}(u) = p y - c - u\). The agent puts in effort and receives \(w(u) = u - u^{*}_{LT, 2}\). His continuation payoff moves to \(u_{l}(u) = u\) when output is low, and to \(u_{h}(u) = u_{l}(u) + k\) when output is high.

An optimal long-term contract starts in the probationary region.

**Proposition 0** follows directly from Biais et al. (2004, 2013). The payoff frontier under the optimal long-term contract consists of three regions, a common feature of dynamic-contracting models with limited liability. In the right, there is a reward region in which the agent puts in effort and receives wage above 0. This region is self-absorbing, and relationship in this region reaches first best. In the middle, there is a probationary region. The agent puts in effort, but his wage is 0—he is motivated entirely through changes in continuation payoffs. Note that \(k\) in (iii.) is the minimal difference in \(u_{h}\) and \(u_{l}\) such that (IC) is satisfied. Finally, there is a punishment region in the left in which randomization occurs, and the relationship is terminated with positive probability.

The optimal long-term contract starts in the probationary region, and the agent is rewarded or punished only when his continuation payoff falls into the reward or the punishment region. The three-region structure (and the associated values of the boundaries) arises because it allows the
reward and punishment to the agent to be delayed as much as possible. This way, the principal reduces the payment to the agent and allows the incentives to be reused.

Despite the similarities to Biais et al. (2004, 2013), there are some notable differences. One is that, in our setup, the principal does not make a output-contingent payment—the rewards are delayed to the beginning of next period as part of the base wage. As a result, the contractual arrangement in our setup looks different from those in Biais et al. (2004, 2013). But as mentioned above, there is a one-to-one relationship between these two setups through the delay of bonus payment to the beginning of the following period.

Another difference is that, in our setup, the agent incurs a cost of effort, but no effort is needed in Biais et al. (2004, 2013). Rather, the agent “shirks” by obtaining a private benefit (of value \( c \)). This implies that the corresponding per-period payoff of the agent is \( c \) lower in our “cost-of-effort” approach than in their “private benefit” approach, leading to differences in mathematical expressions. The boundary of the reward region \( u_{LT}^{\ast} \) \((qc/(p-q))\), in particular, is \( c \) smaller in ours than that in theirs. This follows because once the agent’s payoff reaches \( u_{LT}^{\ast} \), his payoff is \( c \) lower than in our setup in all future periods. Similarly, the value of the left boundary of the probationary region, \( u_{LT,1}^{\ast} \), is \((1-\delta)c\) smaller than the corresponding expression in Biais et al. (2004, 2013). The factor \( 1-\delta \) arises because the difference in payoffs occurs for one period only (since at the left boundary of the probationary region, the agent’s continuation payoffs have the same expressions in the two setups—both equal to 0 for low output and both equal to \( k \) for high output).

4. Optimal relational contract and its implications

4.1. Properties of the optimal relational contract

We now describe the payoff frontier and the optimal relational contract. Let \( f(u) \) be the principal’s highest PPE payoff when the agent’s payoff is \( u \), i.e., \( f(u) = \max\{v' : (u, v') \in \mathcal{E}\} \). Define \( \bar{\pi} = \max\{u : (u, v) \in \mathcal{E}\} \) as the agent’s largest PPE payoff. Note that \( \bar{\pi} \) is finite because the principal cannot make unbounded promises—her continuation payoffs must exceed her outside option \( v \). It then follows from standard arguments that \( \mathcal{E} \) is compact, so \( f \) and \( \bar{\pi} \) are well-defined, and furthermore, \( f \) is concave since a public randomization device is available. As in the case of the optimal long-term contract, for any payoff \((u, f(u))\) on the frontier, its continuation payoffs remain on the frontier. This follows because the principal’s actions are publicly observable, so she is never punished on the equilibrium path.\(^{10}\) This implies that the PPE payoff set and the optimal relational contract are completely determined by the payoff frontier \( f(u) \).

To simplify the description of \( f(u) \), note that, when the agent is asked to put in effort, \( k = (1-\delta)c/\delta(p-q) \) is the smallest difference between the agent’s continuation payoffs \((u_h - u_l)\) so that (IC) binds. When (IC) binds, for an agent with payoff \( u \) who puts in effort and receives \( w \) this period, (PKA) implies that his continuation payoffs are given by \( u_l(u) = L(u) \equiv u/\delta - (1-\delta)(w+qc/(p-q))/\delta \) and \( u_h(u) = L(u) + k \). A property of \( L(u) \)—useful in our discussion below—is that when an agent with payoff \( u \) is asked to put in effort, his continuation payoff following a low output cannot exceed \( L(u) \), and it reaches \( L(u) \) when the wage is \( w \) and \( u_h(u) = L(u) + k \). The proposition below describes the optimal relational contract.

\(^{10}\) When multiple parties take private actions, joint punishments may be necessary; see, for example, Green and Porter (1984), Athey and Bagwell (2001), and the second part of Levin (2003).
Proposition 1. There exist \( u_1^*, u_2^*, w^* \), and \( \delta^* \) that the following holds.

(i.) (Punishment Region) For all \( u \in [u_1, u_1^*] \), \( f(u) \) is linear, and

\[
u_1^* = \begin{cases} 
L^{-1}(u) & \text{if } w < u + (1 - \delta q)c/(\delta(p - q)) \\
(1 - \delta q)c/(p - q) & \text{if } w \geq u + (1 - \delta q)c/(\delta(p - q)).
\end{cases}
\]

There exists \( w^* < u \) such that, if \( w < w^* \), \( (u, f(u)) \) is supported with shirking. The agent is paid \( w \) and has a continuation payoff \( (u - (1 - \delta)w)/\delta \). If \( w \geq w^* \), \( (u, f(u)) \) is supported with no-entry. The agent’s continuation payoff remains at \( u \), so the relationship terminates.

(ii.) (Probationary Region) For all \( u \in [u_1^*, u_2^*] \), \( f(u) = (1 - \delta)(py - w) + \delta[pf(u_2(u)) + (1 - p)f(u_1(u))] \). The agent is paid \( w \) and puts in effort. His continuation payoffs are given by \( u_l(u) = L(u) \leq u \) and \( u_h(u) = L(u) + k \geq u \).

(iii.) (Reward Region) For \( u \in (u_2^*, \bar{u}] \), \( f'(u) = -1 \), and

\[
u_2^* = \begin{cases} 
L^{-1}(u - k) & \text{if } \delta < \delta^* \\
qw +qc/(p-q) & \text{if } \delta \geq \delta^*,
\end{cases}
\]

where \( \delta^* = 1/((1 - p + (p - q))py - w)/(p - q) \). The agent receives \( w(u) = w + (u - u_2^*)/(1 - \delta) \) and puts in effort. His continuation payoffs are given by \( u_l(u) = L(u_2^*) \) and \( u_h(u) = L(u_2^*) + k \). When \( \delta < \delta^* \), the continuation payoffs are unique, and \( u_l(u) = L(u_2^*) < u^*_2 \).

An optimal relational contract starts in the probationary region.

Proposition 1 shows that the optimal relational contract shares many features with the optimal long-term contract in Proposition 0. The payoff frontier again has three regions, and the relationship starts in the probationary region, where the agent puts in effort and receives a wage equal to the wage floor. The agent’s continuation payoff moves to the right when output is high, and it moves to the left when output is low. The agent is rewarded when his continuation payoff moves to the reward region, and he is punished when his continuation payoff moves to the punishment region.

Despite the similarities, there are important differences. In particular, while the reward region is always absorbing in the optimal long-term contract, it is absorbing in the optimal relational contract only when \( \delta \geq \delta^* \). The cutoff \( \delta^* \) has the property that when the agent’s payoff is at \( w +qc/(p-q) \)—corresponding to the reward region’s boundary in the long-term contract \( (qc/(p-q)) \) when \( w = 0 \), the principal’s continuation payoff following a high output is exactly equal to \( v \). In other words, \( \delta^* \) is the smallest discount factor at which the essentially stationary contract—specified in the reward region of the optimal long-term contract—is a relational contract. At \( \delta^* \), the reward that the principal must give out for high output in the stationary contract is equal to her future surplus in the relationship, so the principal’s “non-reneging constraint” just binds. As mentioned above, relational contracts can be thought of as long-term contracts with the extra non-reneging constraints from the principal. When \( \delta \geq \delta^* \), the non-reneging constraints can be made slack, rendering the relational contracts identical to the long-term contracts.\(^{11}\)

When \( \delta < \delta^* \), the reward region is non-absorbing. In particular, there is not enough surplus in the relationship to make the stationary contract mentioned above a relational contract. This

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\(^{11}\) One caveat is that since there are infinitely many nonreneging constraints—one for each history—it is possible for the optimal long-term and relational contracts to differ for all \( \delta \) in general; see, for example, Harris and Holmstrom (1982), Thomas and Worrall (1988), and Li and Matouschek (2013). The key condition for the two to be the same is that the principal’s continuation payoffs must be strictly and uniformly (for all histories) above her outside option whenever she can renege.
implies that, to maintain the reward amount necessary to induce effort, the agent’s continuation payoff following a low output must fall—otherwise his continuation payoff following a high output would be too high, causing the principal’s continuation payoff to fall below \( v \). This makes the reward region non-absorbing, which occurs precisely because the surplus of the relationship constrains the principal’s ability to backload reward. Of course, the principal still would like to backload reward as much as possible. As a result, the value of the boundary the reward region, \( u^*_5 \), is given by \( u^*_5 (u^*_5) = \bar{u} \) (so that the agent’s continuation payoff following a high output is at the maximal equilibrium level).

In addition to making the reward region non-absorbing, the principal’s lack of commitment also affects the set of feasible contractual arrangements. When the principal can commit to a long-term contract, there is no restriction on the payment paths in the reward region as long as these payment paths give the same continuation payoffs to the agent. With optimal relational contracts, the agent’s continuation payoffs, and therefore the payments, are unique when \( \delta < \delta^* \). Even when \( \delta \geq \delta^* \), the principal still has less flexibility in arranging payment paths than she does under the long-term contract because she cannot delay payments so much that her continuation payoff falls below \( v \). In particular, if the agent’s payoff \( u > \bar{u} - k \), where, as pointed out earlier, \( k \) is the minimal difference in the agent’s continuation payoff to induce effort, the agent’s continuation payoff following a low output must fall. This implies that an (essentially) stationary contract cannot be used if the agent’s payoff \( u > \bar{u} - k \), but a stationary contract can always be used in the reward region in the optimal long-term contract.

In addition to the differences in the reward region, Proposition 1 shows that the punishment region differs in two ways. First, unlike that in Proposition 0, there are two ways to punish the agent at \( u \). If the wage floor \( w \) is greater than a cutoff \( w^* \) (that is smaller than \( u \)), the agent is punished via no-entry. The agent’s continuation payoff therefore remains at \( u \), causing termination to occur. If the wage floor \( w \) is smaller than \( w^* < u \), the agent is asked to shirk, and his continuation payoff moves to the right to \( u_c (u) = (u - (1 - \delta) w) / \delta \). If \( u_c (u) \) moves out of the punishment region, the agent will put in effort next period. If \( u_c (u) \) remains in the punishment region, randomization (between \( u \) and \( u^*_2 \)) occurs, and depending on the outcome of the randomization, the agent either puts in effort or shirks again. Figs. 2a and 2b illustrate the payoff frontier corresponding to two different ways of punishment. The figures are generated based on these parameters: \( p = 1/2, q = 1/4, \delta = 0.8, c = 1, y = 20, u = 1, \) and \( v = 5 \), resulting in \( k = 1 \). The only difference between them is that in Fig. 2a, \( w = 1 \) and in Fig. 2b, \( w = 5/9 \).

To see what determines the use of no-entry or shirking, note that when shirking is used, the joint current payoff of the principal and the agent is \( qy \), which is smaller than \( u + v \), the joint payoff when no-entry is used. Shirking therefore creates a larger efficiency loss, and, everything else equal, is the less preferred method of punishment. However, when \( w < u \), shirking has the advantage of allowing the principal to more harshly punish the agent since it gives the agent a payoff lower than his outside option. As \( w \) decreases, shirking becomes a more effective tool for punishment, and eventually dominates no-entry.

Note that the extra punishment possibility arises not because of lack of commitment, but because we consider a wider range of \( u \) and \( y \). When \( u = v = 0 \), the punishment region is identical to that in Proposition 0. The relationship terminates when the agent’s payoff is equal to \( u \). And the boundary of the punishment region \( (u^*_2) \) becomes \( L^{-1} (0) = (1 - \delta) q c / (p - q) \), the smallest payoff the agent must receive to put in effort. When \( w \) is sufficiently smaller than \( u \), however, shirking can also be used in the optimal long-term contract. While shirking can also arise under the optimal long-term contract, lack of commitment affects the conditions under which it will arise, as we will see in the next section.
Fig. 2. (a) PPE payoff frontier with termination. (b) PPE payoff frontier with suspension.
Second, Proposition 1 shows that the value of the punishment region’s right boundary \( u_1^* \) differs from that in Proposition 0. Before discussing the difference, first note that when \( w \) is not too high (relative to \( u \), \( u_1^* = L^{-1} (u) \) has the same expression as that in Proposition 0 (with \( w = u \)). To see why this is the case, recall that, to induce effort from an agent with payoff \( u \), his continuation payoff following a low output cannot exceed \( L (u) \). This expression of \( u_1^* \) therefore reflects the familiar idea that in a repeated relationship, the principal would like to backload punishment as much as possible—since for \( u < L^{-1} (u) \) the agent’s payoff following a low output would fall below his outside option.

When \( w \) is sufficiently high (relative to \( u \)), however, Proposition 1 shows that the boundary is determined via \( u_1^* = u_h (u_1^*) = L (u_1^* + k) \). In this case, the agent’s current payoff may be so high that, even if output is high, his continuation payoff decreases, i.e., \( u_h (u) < u \) for some \( u \). In this case, \( u_1^* \) is the cutoff value such that once the agent’s payoff falls below \( u_1^* \), his continuation payoff will never exceed it, making the punishment region absorbing. Unlike the reward region, whether the punishment region is absorbing or not has no effect on the long-run dynamics of the relationship, as we see in the corollary below.

Corollary 1. The following holds.

(i.) High Surplus and High Wage Floor \((\delta \geq \delta^*, \ w \geq w^*)\). The relationship either terminates or stays in the reward region: \( \lim_{t \to \infty} \Pr (u_t \geq u_1^*) = 1 - \lim_{t \to \infty} \Pr (u_t = u) \in (0, 1) \).

(ii.) High Surplus and Low Wage Floor \((\delta \geq \delta^*, \ w < w^*)\). The relationship ends in the reward region with probability \( I \): \( \lim_{t \to \infty} \Pr (u_t \geq u_2^*) = 1 \).

(iii.) Low Surplus and High Wage Floor \((\delta < \delta^*, \ w \geq w^*)\). The relationship terminates with probability \( I \): \( \lim_{t \to \infty} \Pr (u_t = u) = 1 \).

(iv.) Low Surplus and Low Wage Floor \((\delta < \delta^*, \ w < w^*)\). The relationship never terminates, and the total payoff of the relationship fluctuates: \( \lim_{t \to \infty} u_t + v_t \) does not exist.

The long-run outcomes of the relationship follow directly from Proposition 1. Corollary 1 shows that there is a large variety of long-run outcomes. Depending on the wage floor and the surplus level, the relationship may terminate with probability 0, 1, or somewhere in between. In addition, a surviving relationship may or may not attain efficiency in the long run. In contrast, if the principal can commit, the variety of long-run outcomes are more limited. The relationship always survives with positive probability in the long run (because the reward region is absorbing). Moreover, if the relationship survives in the long run, it attains efficiency. Consequently, the principal’s ability to commit affects the dynamics of the relationship, and we discuss its effects in detail in the next section.

4.2. Implications

We now discuss the implications of the model. To start, note that Proposition 1 has a number of implications on the agent’s pay and turnover dynamics. In terms of pay, it implies that the pay of the worker is initially low and is insensitive to output. This follows because the pay of the worker starts at \( w \)—given the relationship starts in the probationary region—and stays at \( w \) as long as his payoff remains in the probationary region. Only after the agent’s payoff reaches the reward region, his pay is above \( w \), and output affects the agent’s pay next period.

In terms of turnover patterns, Proposition 1 implies that if termination occurs in a high-surplus relationship \((w \geq w^* \text{ and } \delta \geq \delta^*)\), the rate of termination may be inverse-U shaped with respect to the duration of the relationship. To see this, note that at the beginning of the relationship,
the turnover rate is low—and can be 0—since it takes time for the agent’s continuation payoff to reach the punishment region and trigger termination. But in the long run, the termination rate again converges to zero: for a relationship that lasts long enough, it is most likely to be in the (absorbing) reward region and no longer faces termination. It follows that the turnover rate initially increases but decreases eventually, pointing to an inverse-U shaped relationship.

The model’s implications on pay and turnover dynamics are broadly consistent with empirical findings; see, for example, Farber (1994) for a survey on turnover patterns and Rubinstein and Weiss (2006) on wage patterns. It should be mentioned that while there is strong support for inverse-U shaped turnover rate and the upward-sloping wage profile, the evidence is weaker for that pay may be more sensitive to performance over time. In addition, various aspects of wage and turnover patterns can be explained by other theories, such as learning and human capital accumulation. By leaving out these elements, our model does not aim at fully accounting for wage and turnover patterns since it generates less realistic patterns—for example, wages can decrease. Rather, our model captures a mechanism that may be useful for understanding—simultaneously—wage and turnover patterns.

In addition, our model implies that commitment power affects the dynamics of the relationship. To see this, we compare the optimal long-term and relational contract. Recall that we described the optimal long-term contract for \( u = w = 0 \) in Proposition 0. When we consider a wide range of \( u \) and \( w \), the payoff frontier \( f_{LT} \) again has three regions, divided by the two cut-offs \( u_{LT,1}^* \) and \( u_{LT,2}^* \). But as mentioned above, unlike that in Proposition 0, where punishment always takes the form of no-entry, now there exists a cutoff wage floor \( w_{LT}^* < u \)—just as in the optimal relational contract—such that if \( w \leq w_{LT}^* \), \( (u, f_{LT} (u)) \) is supported with shirking, and if \( w > w_{LT}^* \), \( (u, f_{LT} (u)) \) is supported with no-entry.

Despite the similarities, the payoff frontiers—with and without commitment—also have significant differences. Only when the relationship has high surplus (\( \delta \geq \delta^* \)), the principal’s inability to commit does not affect the PPE payoff frontier nor the dynamics of the relationship. When \( \delta < \delta^* \), there are differences in the cutoffs that determine the three regions of the payoff frontier and the use of termination—\( u_{LT,1}^* \), \( u_{LT,2}^* \) and \( w_{LT}^* \) for long-term contract and \( u_{R,1}^* \), \( u_{R,2}^* \) and \( w_{R}^* \) for the relational contract, where we added the subscript \( R \) to make the distinction clearer. The differences in the cutoffs also lead to differences in the dynamics of the relationship, which are summarized in Proposition 2.

**Proposition 2.** If \( \delta < \delta^* \), then \( w_R^* < w_{LT}^* \), \( u_{R,2}^* < u_{LT,2}^* \) and the following holds.

(i.) Relationships under the optimal relational contracts are less likely to survive in the long run.

(ii.) Among the surviving relationships, total surplus and expected wages are lower in the optimal relational contract.

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12 Hashimoto (1979) finds that the bonus to wage ratio is increasing with experience in Japanese firms. Gibbons and Murphy (1992) show that the pay of older CEOs is more sensitive to stock-market performance. Gompers and Lerner (1999) document that the sensitivity of pay to performance is smaller for newer venture capitalists. Misra et al. (2005) find that the salary to total compensation ratio is decreasing with salesperson seniority. However, Kahn and Sherer (1990) find that bonuses are more sensitive to performance for less senior managers.

13 The turnover patterns can also be explained by learning models such as Jovanovic (1979). The increasing wage profile can result from any models of human-capital accumulation. The increasing sensitivity of pay to performance is often explained by career-concern models; see, for example, Gibbons and Murphy (1992) and Gompers and Lerner (1999).
(iii.) The total payoff of the relationship fluctuates in the optimal relational contract for \( w \leq w_R^* \), but converges with probability 1 under the optimal long-term contracts.

Part (i) of the proposition shows that lack of commitment makes the relationship less likely to survive in the long run.\(^{14}\) There are two reasons for this. First, lack of commitment constrains the size of monetary reward by the amount of future surplus. When \( \delta < \delta^* \), the reward alone is not enough to motivate the agent, and the principal must always keep the threat of termination. In contrast, when the principal can commit, the size of the reward is no longer constrained, and by the first time the principal pays out the reward, the relationship never terminates.

Second, lack of commitment reduces the value of the relationship, making termination a less costly way to punish. When \( w \in (w_R^*, w_{LT}^*) \), the long-term contract uses shirking as punishment and the relational contract uses termination. In this case, the relationship terminates with probability 1 under relational contract and never terminates under long-term contract. When \( w \geq w_{LT}^* \), once again the relationship terminates with probability 1 under the relational contract but terminates with a probability between zero and one under long-term contract. If we consider a population of relationships with different \( \delta^* \) and that \( \delta < \delta^* \) for some of these relationships and \( \delta \geq \delta^* \) for the other relationships, then given a distribution of relationships, lack of commitment still leads to a higher probability of termination.

In addition to making the relationship less likely to survive, lack of commitment also affects the surviving relationships, as parts (ii) and (iii) of the proposition indicate. Under the optimal long-term contract, once the relationship falls into the reward region, it never leaves, and the relationship is efficient. In contrast, the reward region is non-absorbing under the relational contract when \( \delta < \delta^* \). As a result, the agent’s continuation payoff will fall into the punishment region following sufficiently many consecutive low outputs, which implies that the agent’s payoff cycles between the reward and punishment regions. Consequently, the per-period total payoff of the relationship fluctuates between \( py - c \) (when the agent puts in effort) and \( qy \) (when shirking is used). In addition, the agent’s pay also fluctuates.

Direct evidence of these implications of lack of commitment is difficult to obtain because firms with different levels of commitment power typically differ in other aspects of the production environment, making it difficult to attribute observed differences to the variations in commitment power. One setting in which firms have a similar production environment but differ in their commitment power is the franchise industry, where company-owned outlets might have more commitment power than franchisee-owned ones.\(^{15}\) In addition, many workers in this industry receive minimum wages, suggesting that limited-liability constraints are relevant.\(^{16}\)

\(^{14}\) In addition to the long-run results, it can be shown that, when punishment takes the form of termination, the agent’s payoff under the optimal relational contract is smaller, implying that, under the optimal relational contract, it takes (weakly) fewer consecutive low outputs to trigger positive probability of termination. See Corollary A2 in the online appendix for a formal statement and proof.

\(^{15}\) Our assumption that the corporation has more commitment power than the franchisee is consistent with the allegation against a McDonald’s franchisee for “failing to pay overtime, failure to pay minimum wage, and reducing pay by falsely recording information on timecards” and judgment against a Papa John’s franchisee for shortchanging workers on their wages. (See “McDonald’s Overtime Lawsuit Granted Class Action Status,” www.lawyersandsettlements.com on July 28, 2016 and “Papa John’s franchise owner will pay $500G for ripping off more than 200 NYC workers,” www.mydailynews.com, August 22, 2016).

\(^{16}\) Of course, company-owned and franchises-owned outlets may differ in other dimensions—notably, the franchisee, as the owner of the outlet, has a stronger incentive to monitor the workers vis-à-vis the managers in company-owned outlets. This also causes differences in wage and turnover patterns.
Two studies examining these differences are Krueger (1991) for fast food restaurants and Freedman and Kosová (2014) for hotels. Krueger (1991) found that the starting pay of lower-level managers is about the same at company-owned and franchisee-owned restaurants, but their pay increases more rapidly at company-owned restaurants. Freedman and Kosová (2014) found that, for an occupation at a hotel, the gap between the highest wage paid and starting wage is higher at company-managed properties. These findings are consistent with our predictions that company-owned outlets can better backload the compensation of their workforce, resulting in higher wages in the long run, and relatedly, steeper wage-tenure profiles.

Our theory also predicts that a lack of commitment reduces the survival probability of the relationship. The evidence on this is more scant. Krueger (1991) found that lower-level managers have a half-year longer tenure at company-owned stores. For the crew workers, whose activities are more routine, there is no difference in tenure. Freedman and Kosová (2014) do not have data on worker tenure.

5. Conclusion

This paper studies a model of relational contracts with limited liability. We characterize the optimal relational contract and explore the consequence of non-commitment. Our theory predicts upward trends of pay and pay sensitivity but a potentially inverse-U-shaped turnover rate with respect to the duration of the relationship. Compared to the optimal long-term contract, the principal’s inability to commit makes the relationship less likely to survive, and changes the long-run outcomes of the surviving relationships. Whereas the total surplus of the surviving relationships under optimal long-term contracts always converge to the first best in the long run, it can fluctuate under the optimal relational contracts.

In the supplementary online appendix, we consider three extensions of the model, allowing the output to be multi-valued, the principal to be more patient than the agent, and the effort level to be continuous. Our main results are robust to these extensions: the agent’s starting wage is always equal to the wage floor, and his continuation payoff moves to the right when output is sufficiently high and moves to the left otherwise. In particular, when effort level is continuous, termination can still occur, for example, when sufficiently low effort level also implies that high output is very unlikely.

The extensions also lead to some differences in the dynamics of the relationship. When output has multiple values, for example, the agent’s continuation payoff in the probationary region is no longer binary, but is rather increasing in the output level. In addition, even if the reward region is absorbing (so that first best can be reached), the rewards are more frequent but are smaller in size under the optimal relational contract. The difference in the contractual arrangement arises again because the size of the reward is constrained by the principal’s future surplus of the relationship.

When the principal is more patient than the agent, there is a new case in which lack of commitment makes the relationship less likely to survive. Note that when the principal is more patient,

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To the extent that the work of the crew is routine, their moral hazard problem is close to our model with $p = 1$. When $p = 1$, our model implies that there is no equilibrium turnover and the relationship is efficient (when it can be sustained). In this case, commitment does not affect employment dynamics. We discussed the case of $p = 1$ in an earlier version of the paper, and the analysis is available upon request.
she has an incentive to frontload the payment to the agent.\footnote{Opp and Zhu (2015) study how this incentive to frontload affects the repeated relationship in a general environment. They show that the relationship may fluctuate even if the environment has no uncertainty.} The benefit of frontloading can be so large that, for some parameter values, the optimal long-term contract is essentially stationary—the agent is rewarded immediately (at the beginning of next period) for each high output. For the same parameter values, however, the optimal relational contract can be non-stationary, which occurs when the reward amount in the stationary contract exceeds the principal’s future surplus. In this case, the optimal relational contract not only is nonstationary but also terminates with probability 1 in the long run. In contrast, the optimal long-term contract is stationary.

A possible extension in the future is to consider multiple firms and workers. In this case, unlike the current model, the optimal relational contract can become renegotiation-proof.\footnote{The lack of renegotiation-proofness is a common feature both for optimal relational contract and optimal long-term contracts; see, for example, Thomas and Worrall (1994), Gromb (1999), Quadrini (2004), DeMarzo and Fishman (2007), and Hörn and Samuelson (2016) for discussion.} When the firm can costlessly find a replacement in the case of agent termination, the punishment region in the PPE payoff becomes a flat line instead of an upward-sloping one. When the agent’s continuation payoff falls into the punishment region, the principal is indifferent between keeping the current agent and finding a new one, and, hence, no renegotiation takes place.

One possible consequence of the extension is that multiple equilibrium turnover patterns can arise.\footnote{MacLeod and Malcomson (1989, 1998) consider relational contracts with multiple workers and firms when information is public. They show that multiple equilibria arise, although the multiplicity does not apply to the turnover rates but rather to the different ways to divide the surplus of the relationship.} The reason is that turnover patterns affect the outside options, which in turn affect the surplus in the relationship, which affects the turnover pattern. Consider, for example, an economy in which vacant firms and unemployed workers match randomly. When it becomes easier to form new employment relationships, the surplus in the existing relationship is lowered due to the higher outside options. Our analysis implies that the relationships are more likely to dissolve when the surplus is lower. This increases the number of vacant firms and unemployed workers, making it even easier to form new employment relationships. Such multiplicity may help shed light on the large cross-country differences in employment patterns.

**Appendix A**

The proof of Proposition 0 can be adapted directly from Bi ais et al. (2004, 2013) and is omitted.

**Proof of Proposition 1.** To prove Proposition 1, it is convenient to first establish a property of the payoff frontier.

**Property A.** $f'_+(u) \geq -1$ for all $u \in [u, \bar{u}]$. In addition, $f'(u) = -1$ if $w(u) > w_*$, where $w(u)$ is the wage associated with $(u, f(u))$.

To prove $f'_+(u) \geq -1$, it suffices to show that $f'_+(u) \geq -1$ when $(u, f(u))$ is supported with a pure action. First, suppose $(u, f(u))$ is supported by either shirking or effort. In either case, let $w(u)$ be the associated wage. Now consider an alternative strategy profile with the same action and continuation payoffs but the wage is increased to $w(u) + \varepsilon$ for some small $\varepsilon > 0$. The
alternative strategy profile gives a payoff pair of \((u + (1 - \delta) \varepsilon, f(u) - (1 - \delta) \varepsilon)\), and for small enough \(\varepsilon\), it satisfies all the constraints in Section 3.1, and is therefore a PPE payoff. It follows that \(f(u) - (1 - \delta) \varepsilon \leq f(u + (1 - \delta) \varepsilon)\), and sending \(\varepsilon\) to 0, we have that \(f'_+(u) \geq -1\). Next, suppose \((u, f(u))\) is supported by exit. In this case, the promise-keeping constraint of the agent and the concavity of \(f\) implies that there exists \(u' \in (u, \tilde{u})\) such that \((u', f(u'))\) is supported with either shirking or effort. By the argument above, we have \(f'_+(u') \geq -1\), and since \(u' > u\) and \(f\) is concave, this implies that \(f'_+(u) \geq f'_+(u') \geq -1\). This proves that \(f'_+(u) \geq -1\) for all \(u \in [u, \tilde{u}]\).

Next, we show that if \(w(u) > w\), then \(f'(u) = -1\). To see this, notice that if \(w(u) > w\), then the same argument above can be applied to the case in which \(\varepsilon < 0\), and this proves that \(f'_+(u) \leq -1\). Now since \(f\) is concave, we have \(f'_+(u) \leq f'_+(u) \leq -1\). This, together with \(f'_+(u) \geq -1\), implies that \(f'_+(u) = f'_+(u) = -1\). This proves Property A.

Part (i): We first show that exists \(u^*_1\) such that \(f(u)\) is linear in \([u, u^*_1]\). This follows from the following two steps.

**Step 1:** If \((u, f(u))\) is supported with exit for some \(u\), then \(f(u) = v\). In addition, there exists \(u^*_1 \geq u\) such that \(f(\cdot)\) is linear in \([u, u^*_1]\). To see this, suppose \((u, f(u))\) is supported with exit. The agent’s promise-keeping constraint implies \(u_x(u) = (u - (1 - \delta) u) / \delta\). For all \(z \in [u, \tilde{u}]\), define \(f_x(z) = (1 - \delta) v + \delta f(u_x(z))\). This is the highest equilibrium payoff for the principal that is supported by exit and gives the agent a payoff of \(z\). Since \((u, f(u))\) is supported with exit, we have \(f(u) = f_x(u)\). Also notice that \(f_x(u) = v\) since \(u_x(u) = u\).

Now for \(z > u\), we have \(u_x(z) = (z - (1 - \delta) u) / \delta > z\). Since \(f\) is concave, we have \(f'_x(z) = f'_x(u_x(z)) \leq f'_x(z) \leq f'_x(z)\). Consequently,

\[
    f(u) = f(u) - \int_u^u f'_x(z) \, dz \leq f_x(u) - \int_u^u f'_x(z) \, dz = f_x(u),
\]

where recall that \(f_x(u) = f(u)\).

Since \(f\) is the PPE payoff frontier, we must also have \(f(u) \geq f_x(u)\). It then follows that \(f(u) = f_x(u) = v\).

Next, since \(f(u) = f_x(u)\), the argument above then implies that \(f'_x(u_x(z)) = f'_x(u_x(z)) = f'_x(u_x(z))\) for almost all \(z \leq u\). The concavity of \(f\) then implies that \(f\) is linear in \([u, u_x(u)]\). Let \(u^*_1\) be the right end of this line segment. This proves Step 1.

**Step 2:** We now show that if \((u, f(u))\) is supported with shirking for some \(u\), then \(w(u) = w\) and \(w < u\). In addition, there exists \(u^*_2\) such that \(f(u)\) is linear in \([u, u^*_2]\).

We first show that, if \((u, f(u))\) is supported with shirking, then \(w(u) = w\). Suppose to the contrary that \(w(u) > w\). Then \(f'_+(u) = -1\) by Property A. In addition, we have \(f'_+(u_x(u)) = -1\). Otherwise, we have \(f'_+(u_x(u)) < -1\) (since \(f'(u_x(u)) \geq -1\) by Property A). But then consider an alternative strategy profile that is supported with shirking and has \(\tilde{w} = w(u - \delta \varepsilon)\) and \(\tilde{u}_x = u_x(u) + (1 - \delta) \varepsilon\). For small enough \(\varepsilon > 0\), this strategy profile is a PPE. It gives the agent a payoff of \(u\). And since \(f'_+(u_x(u)) < -1\), it gives the principal a payoff that exceeds \(f(u)\). This contradicts the definition of \(f\) (as the PPE payoff frontier).

Given that \(f'_+(u_x(u)) = -1\) and \(f'_+(u) = -1\), it follows \(u \) and \(u_x(u)\) both lie on the same line segment with slope \(-1\), and therefore, \(u + f(u) = u_x(u) + f(u_x(u))\). Adding promise-keeping the constraints of the principal and the agent, we then have that
\[ u + f(u) = (1 - \delta) qy + \delta (u_s(u) + f(u_s(u))) \]
\[ = (1 - \delta) qy + \delta (u + f(u)) . \]

This implies that \( u + f(u) = qy < u + v \), which is a contradiction. This proves that \( w(u) = w \).

Given \( w(u) = w \), the agent’s promise-keeping constraint then implies \( u_s(u) = (u - (1 - \delta) w) / \delta \). Define
\[ f_s(u) = (1 - \delta) (qy - w) + \delta f(u_s(u)) , \]
which is the principal’s maximum PPE payoff that gives the agent \( u \) and that is supported with shirking and \( w(u) = w \).

Next, we show that, if \( f_s(u) = f(u) \) for some \( u \), then \( u > w \). To do this, we first show that if \( f_s(u) = f(u) \) for some \( u \), then \( u > w \). Suppose to the contrary that \( u \leq w \). Notice that for all \( u' \leq w \), \( u_s(u') \leq u' \) by (PKA). This implies that \( f_{s+}'(u') = f_{s+}'(u_s(u')) \geq f_{s+}'(u') \) by the concavity of \( f \). It follows that
\[ f(w) \geq f_s(w) = f_s(u) + \int_u^w f_{s+}'(z)dz \geq f(u) + \int_u^w f_s'(z)dz = f(w) . \]

We then have \( f(w) = f_s(w) = (1 - \delta) (qy - w) + \delta f(w) = qy - w \). Since \( u < u \leq w \), this implies that \( f(w) < qy - u < v \), which is a contradiction. This proves that \( u > w \).

We now show that \( u > w \). To see this, notice that \( u_s(u') > u' \) for all \( u' \in [w, u] \), so \( f_{s+}'(u') = f_{s}'(u_s(u')) \leq f_{s+}'(u') \). Now suppose to the contrary that \( w \geq u \), then
\[ f_s(u) = f_s(w) + \int_u^w f_{s+}'(z)dz < f(w) + \int_u^w f_{s}'(z)dz = f(u) , \]
where the inequality is strict because \( f_s(w) < f(w) \). (Otherwise, \( f_s(w) = f(w) = qy - w \leq qy - u < v \), which is a contradiction.) But this contradicts the assumption that \( f_s(u) = f(u) \), and therefore, we cannot have \( w \geq u \). This proves \( u > w \).

Given \( u > w \), we have \( f_{s+}'(u') = f_{s}'(u_s(u')) \leq f_{s+}'(u') \) for all \( u' \in [u, u] \), so
\[ f_s(u) = f_s(u) - \int_u^w f_{s+}'(z)dz \geq f(u) - \int_u^w f_{s}'(z)dz = f(u) . \]

This chain of inequalities corresponds to that in Step 1, and therefore, \( f \) is linear in \([u, u_s(u)]\). Let \( u^*_s \) be the right end of this line segment. This proves Step 2.

We now prove Part (i) using Step 1 and 2. Note \( (u, f(u)) \) is an extremal point of the PPE payoff frontier, so it is supported by a pure action. In addition, \( (u, f(u)) \) cannot be supported by effort (because otherwise it would violate Condition (LLB)). Consequently, \( (u, f(u)) \) is supported by either no-entry or shirking. Step 1 and 2 imply that, if \( f(u) = v \), then \( (u, f(u)) \) is supported with exit, and if \( f(u) > v \), it is supported with shirking. Let \( u^*_1 = u^*_s \) in the former case and let \( u^*_1 = u^*_s \) in the later case. This proves that \( f \) is linear in \([u, u^*_s] \). Also note that \( f(u) \) is weakly decreasing in \( w \), use \( f(u, w) \) to indicate the dependence of \( f \) on \( w \). It follows that if \( f(u, w) > v \) then \( (u, f(u)) \) is supported with shirking and the agent is paid \( w \).
Finally, we determine $u^*_1$. Note that $L(u^*_1) + k = u^*_1$ when $w = u + (1 - \delta q) c / (\delta (p - q))$. In addition, the self-enforcing condition of the agent implies that $L(u^*_1) \geq u$. It then suffices to show that either $L(u^*_1) = u$ or $L(u^*_1) + k = u^*_1$. Now suppose $L(u^*_1) > u$ and $L(u^*_1) + k \neq u^*_1$. If $L(u^*_1) + k = u_h(u^*_1) < u^*_1$, then $u_l(u^*_1) < u_h(u^*_2) < u^*_1$. Let $s$ be the slope of the payoff frontier in the punishment region. Consider a strategy profile with $\hat{w} = w(u^*_1)$, $\hat{u}_l = u_l(u^*_1) + \varepsilon$, $\hat{u}_h = u_h(u^*_1) + \varepsilon$. For small enough $\varepsilon > 0$, this strategy profile is a PPE and generates a payoff of $(u^*_1 + \varepsilon, f(u^*_1) + s\delta\varepsilon)$, contradicting that $u^*_1$ is the right end of the line segment. Now if $L(u^*_1) + k > u^*_1$, then $f_\hat{w}(u^*_1)) < s$ by the definition of $u^*_1$. Consider a strategy profile with $\hat{w} = w(u^*_1)$, $\hat{u}_l = u_l(u^*_1) - \varepsilon$, $\hat{u}_h = u_h(u^*_1) - \varepsilon$. For small enough $\varepsilon > 0$, this strategy profile is a PPE (because $L(u^*_1) = u_l(u^*_1) > u$) and gives the agent an payoff of $\hat{u} = u^*_1 - \delta s\varepsilon$. The principal’s payoff is given by

$$\hat{v} = (1 - \delta)(py - \hat{w}) + \delta((1 - p) f(\hat{u}_l) + pf(\hat{u}_h))$$

$$= f(u^*_1) + \delta((1 - p) (f(\hat{u}_1) - f(u_l))) + \varepsilon(\hat{u}_h - f(u_h)))$$

$$> f(u^*_1) - s\delta\varepsilon,$$

contradicting that $f$ is the payoff frontier. This proves that either $L(u^*_1) = u$ or $L(u^*_1) + k = u^*_1$. This proves part (i).

Part (ii): Note that by part (i), for all $u \geq u^*_1$, $(u, f(u))$ is not supported by either no-entry or shirking. In addition, by Property A, $f'(u) = -1$ if $w(u) > w$. Now define $u^*_2 \equiv \inf\{u : w(u) > w\}$. It follows that for all $u \in [u^*_1, u^*_2]$, if $(u, f(u))$ is supported by a pure action, it is supported by effort and $w(u) = w$. Next, suppose $(u, f(u))$ is supported by randomization so that $(u, f(u)) = \rho(u_l, f(u_l)) + (1 - \rho)(u_2, f(u_2))$ for some $\rho \in (0, 1)$ and $(\hat{u}_i, f(\hat{u}_i))$, where $(\hat{u}_i, f(\hat{u}_i))$, $i = 1, 2$, are both supported with pure actions. Because $f$ is concave, we can assume without loss of generality that $u^*_1 \leq \hat{u}_1 < \hat{u}_2 \leq u^*_2$. Consequently, both $(\hat{u}_1, f(\hat{u}_1))$ and $(\hat{u}_2, f(\hat{u}_2))$ are supported with effort and $w(\hat{u}_i) = w$, $i = 1, 2$. Now suppose $(\hat{u}_i, f(\hat{u}_i))$, $i = 1, 2$ are associated with continuation payoffs $(u_l, f(u_l))$ and $(u_h, f(u_h))$. Consider an alternative strategy profile with first-period wage $w$ and continuation payoffs $(\hat{u}_i, \hat{v}_i)$ and $(\hat{u}_h, \hat{v}_h)$, where $\hat{u}_i = wu_l + (1 - \rho)u_2$, $\hat{v}_i = \rho f(u_l) + (1 - \rho) f(u_2)$, and define $\hat{u}_h$ and $\hat{v}_h$ analogously. It follows from the promise keeping constraints (PKP) and (PKA) that under this alternative strategy profile the payoffs are given by $\hat{u} = \rho u_1 + (1 - \rho)u_2 = u$ and $\hat{v} = \rho (\hat{u}_1, f(\hat{u}_1)) + (1 - \rho)(\hat{u}_2, f(\hat{u}_2)) = f(u)$. Moreover, it can be checked that the new strategy profile satisfies all of the constraints in Section 3.1, so it is a PPE. This implies that $(u, f(u))$ can be supported with effort and $w(u) = w$. In addition, the concavity of $f$ implies that (IC)$_A$ binds, i.e., $u_h(u) - u_l(u) = k$. The functional equation on $f$ then follows immediately.

Part (iii): For any $u \geq u^*_2$, Property A implies that $f(u) = f(u^*_2) + u^*_2 - u$. Now let $u^*_2,i$ and $u^*_2,h$ be the agent’s continuation payoffs associated with $(u^*_2, f(u^*_2))$ for all $u \geq u^*_2$, consider a strategy profile that specifies effort, with $w(u) = w + (u - u^*_2)/(1 - \delta)$ and continuation payoffs $(u^*_2,i, f(u^*_2))$, $i = l, h$. It can be checked that this strategy profile satisfies all of the constraints in Section 3.1, so it is a PPE. In addition, it gives the agent a payoff of $u$ and the principal a payoff of $f(u)$. Consequently, the payoff frontier can be supported by this strategy profile.

To determine $u^*_2$, there are two possibilities. First, when $\delta \geq \delta^*$, it can be checked that the compensation scheme in the proposition leads to a PPE and reaches first best. It then follows that $u^*_2 \leq w + c / \delta(p - q)$. $f(u) = py - c - u$ for $u \geq u^*_2$. In addition, we can assume that (IC) binds by the concavity of $f$. (PKA) then implies that $u_l(u) = L(u)$ and $u_h(u) = L(u) + k$. 


Now suppose to the contrary that \( u^*_2 < w + c / [\delta(p - q)] \). It follows that \( L(u^*_2) < u^*_2 \), and therefore, \( L(u^*_2) + f(L(u^*_2)) < py - c \) since \( u^*_2 \) is the smallest \( u \) that reaches first best. But this then implies that \( u^*_2 + f(u^*_2) < py - c \), a contradiction. Therefore, \( u^*_2 = L(u^*_2) = w + qc / (p - q) \) when \( \delta \geq \delta^* \).

Second, when \( \delta < \delta^* \), it is clear that \( u^*_2 < w + qc / (p - q) \) (because otherwise we reach first best by the above, contradicting \( \delta < \delta^* \)). It then follows that \( L(u^*_2) < u^*_2 \). Recall that \( (u^*_2, f(u^*_2)) \) is supported with effort with first-period wage \( w \). Denote its associated continuation payoff as \( (u_l, u_h, f(u_l), f(u_h)) \), where we may assume \( u_l = L(u^*_2) \) and \( u_h = L(u^*_2) + k \). Now suppose to the contrary that \( u_h < \bar{u} \). Now consider an alternative strategy profile with the same first-period wage \( w \) but in which the continuation payoffs are given by \( (\hat{u}_l, \hat{u}_h, f(\hat{u}_l), f(\hat{u}_h)) \), where \( \hat{u}_l = u_l + \epsilon \) and \( \hat{u}_h = u_h + \epsilon \) for \( \epsilon > 0 \). (PKA) then implies that the agent’s payoff is \( \hat{u} = u^*_2 + \delta \epsilon \). In addition, (PKP) implies that under this strategy profile the principal’s payoff is given by

\[
\hat{v} = (1 - \delta)(py - w) + \delta((1 - p)f(\hat{u}_l) + p f(\hat{u}_h))
= f(u^*_2) + \delta((1 - p)(f(\hat{u}_l) - f(u_l)) + p (f(\hat{u}_h) - f(u_h)))
\geq f(u^*_2) - \delta \epsilon,
\]

where the strict inequality follows because \( f(\hat{u}_l) - f(u_l) > -\epsilon \) since \( u_l < u^*_2 \). Note that the only constraint that new strategy profile tightens is \( (\text{SE}_h) \). But since \( u_h < \bar{u}, (\text{SE}_h) \) is satisfied for small enough \( \epsilon \), making the new strategy profile a PPE. But this implies \( \hat{v} > f(u^*_2) - \delta \epsilon = f(\hat{u}) \), contradicting the definition of \( f \). This shows that \( L(u^*_2) + k = \bar{u} \) for \( \delta < \delta^* \). This gives the expression of \( u^*_2 \).

Next, we prove the uniqueness, i.e., for all \( u > u^*_2 \), we must have \( u_l = L(u^*_2) \) and \( u_h = L(u^*_2) + k = \bar{u} \) when \( \delta < \delta^* \). Suppose this is not the case, (IC)_A then implies \( u_l < L(u^*_2) \) since \( u_h \leq L(u^*_2) + k = \bar{u} \). Recall also that \( f^\prime_+ (L(u^*_2)) > -1 \). Now if (IC)_A is binding, we have \( u_h = u_l + k < \bar{u} \). Consider an alternative strategy profile with the same first-period wage \( w \) but in which the continuation payoffs are given by \( (\hat{u}_l, \hat{u}_h, f(\hat{u}_l), f(\hat{u}_h)) \), where \( \hat{u}_l = u_l + \epsilon \) and \( \hat{u}_h = u_h + \epsilon \) for \( \epsilon > 0 \). Sending \( \epsilon \) to 0, we then have, by the definition of \( f \),

\[
f^\prime_+ (u^*_2) \geq (1 - p) f^\prime_+ (L(u^*_2)) + p (-1) > -1,
\]

which is a contradiction.

Next, if (IC)_A is slack, we have \( u_h > u_l + k \). Consider an alternative strategy profile with the same first-period wage \( w \) and \( u_h \), but in which \( \hat{u}_l = f(\hat{u}_l) \). Sending \( \epsilon \) to 0, we then have

\[
f^\prime_+ (u^*_2) \geq f^\prime_+ (L(u^*_2)) > -1,
\]

which is again a contradiction. This shows that we must have \( u_l = L(u^*_2) \) and \( u_h = L(u^*_2) + k = \bar{u} \), and (PKA) then implies that the associated wage \( w \) is also unique. This completes the proof of part (iii).

Finally, to see that the optimal relational contract starts in the probationary region, note that by part (i), \( f^\prime_+ (u) > 0 \) for \( u < u^*_2 \) and by part (iii), \( f^\prime_+ (u) < 0 \) for \( u > u^*_2 \), so the payoff frontier is maximized in \([u^*_1, u^*_2] \), and therefore, the agent’s first period payoff \( u \in [u^*_1, u^*_2] \). This completes the proof. \( \Box \)

**Proof of Corollary 1.** The long-run outcomes in the corollary follow directly from Proposition 1 with two caveats. First, we need to rule out that \( u_l = u^*_2 \) when \( \delta \geq \delta^* \), which would imply that the relationship is efficient and \( u_t \geq u^*_2 \) for all \( t \). Second, we need to rule out that \( u_1 = u^*_1 \) when \( u^*_1 = L(u^*_1) + k \), which would imply that \( u_t \leq u^*_1 \) for all \( t \). To rule out the first case, note that
if $u_1 = u^*_2$ when $\delta \geq \delta^*$, $u_l(\underline{u}^*_2) = u^*_2$ and $u_h(\overline{u}^*_2) = u^*_2 + k$. Consider a strategy profile with $\hat{w} = w(\underline{u}^*_2), \hat{u}_l = u_l(\underline{u}^*_2) - \varepsilon, \hat{u}_h = u_h(\overline{u}^*_2) - \varepsilon$. This gives the agent payoff $\hat{u} = u^*_2 - \delta\varepsilon$ and the principal a payoff

$$\hat{v} = (1 - \delta)(py - w(\underline{u}^*_2)) + \delta[pf(u^*_2 + k - \varepsilon) + (1 - p)f(u^*_2 - \varepsilon)].$$

Moreover, for small enough $\varepsilon > 0$, this strategy profile is a PPE, so

$$\hat{v} = f(u^*_2) + \delta[p\varepsilon + (1 - p)(f(u^*_2 + k - \varepsilon)) - f(u^*_2)] \leq f(\hat{u}) = f(u^*_2 - 2\delta\varepsilon).$$

Sending $\varepsilon$ to 0, the inequality above implies that

$$f'(u^*_2) \leq -1.$$

Since $f'(u^*_2) = -1$ and $f$ is concave, this implies that $f'(u^*_2) = -1$, and as a consequence, there exists $u < u^*_2$ such that $f(u) > f(u^*_2)$. In other words, $u_1 < u^*_2$.

To rule out the second case, let $s$ be the slope of $f$ between $u$ and $u^*_1$. Note that if $u_1 = u^*_2$ when $u^*_2 = L(u^*_1) + k$, we have $u_l(u^*_2) = u^*_1 - k$ and $u_h(u^*_2) = u^*_2$. A similar argument as above can then show that $f'(u^*_1) = s$, and therefore, $u_1 > u^*_1$. This finishes the proof. \(\Box\)

**Proof of Proposition 2.** The payoff frontier of $f_{LT}$ can be characterized in a similar way as $f_R$, and the frontier again has three regions. The key distinction is that $u^*_2 = \hat{w} + qc/(p - q)$ for all $\delta$ under the optimal long-term contract, and the reward region is absorbing under the long-term contract. Now to see that $w^*_R < w^*_LT$, notice that when $\delta < \delta^*$, $f_{LT}(u) > f_R(u)$ for all $u \in (\underline{u}, \overline{u}]$. By the definition of $w^*_LT, w^*_R$, and the fact that $f_{LT}(u_*(u)) > f_R(u_*(u))$,

$$(1 - \delta)(qy - \hat{w}^*_R) f_R(u_*(u)) = \hat{v} = (1 - \delta)(qy - \hat{w}^*_LT) f_{LT}(u_*(u)) \geq (1 - \delta)(qy - \hat{w}^*_LT) f_R(u_*(u)).$$

It follows that $w^*_R < w^*_LT$.

Now when $\hat{w} > w^*_LT(> \hat{w}^*_R)$, under both the optimal relational contract and long-term contract, the relationship is terminated with a positive probability when $u$ falls below $u^*_1$. Under the optimal relational contract, a sufficiently long sequence of low outputs (which happens with probability 1 in the long run) will lead $u$ to fall below $u^*_1$, and, therefore, the relationship terminates with probability 1. Under the optimal long-term contract, there is positive probability that $u$ falls into the reward region $(u \geq u^*_1)$, after which the relationship is efficient. When $\hat{w} \in [w^*_R, w^*_LT]$, following the same logic as above, the optimal relational contract terminates with probability 1. In contrast, since the optimal long-term contract is punished with shirking, it never terminates. In addition, a sufficiently long sequence of high outputs (which happens with probability 1 in the long run) will lead $u$ to fall above $u^*_2$, after which the relationship is efficient. When $\hat{w} < w^*_R$, shirking is used for both the optimal long-term and relational contract. In this case, again the same logic as above shows that the relationship under the optimal long-term contract becomes efficient (when $u \geq u^*_2$) with probability 1. Under the relational contract, the dynamics in Proposition 1 makes it clear that $\liminf_l(u_t) = \underline{u}$ and $\limsup_l(u_t) = \overline{u}$. Parts (i) to (iii) follow from the above directly. \(\Box\)

**Appendix B. Supplementary material**

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.jet.2017.02.006.
References


